Mathematical ELEMENTS, In III Parts.

The firſt, being a Diſcourſe of PRACTICAL GEOMETRY, The three Parts of continued Quantity LINES, PLANES, and SOLIDS.

The ſecond, A Deſcription and Uſe of the Cœleſtial and Terreſtrial

GLOBES.

The third, The Delineation of the Globe upon the Plain of any Great Circle, according to the Stereographick, or Circular projection.

By JOHN NEWTON. M. A.

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TO THE

RIGHT WORSHIPFUL

ROGER NORWICH,

ESQUIRE.

Honoured Sir,

I Have not therefore presented you with this small Treatise, that you should defend me from the barkings of the Envious, for then I should put you upon impossibilities, and betray mine own weakness, in being affraid where no fear is, or where there is nothing that ought to be feared. But seeing most men are wont to Dedicate their Labours chiefly to those, to whom they are most obliged, that I may not shew my self ungrateful for the favours I have received from You, I have taken this opportunity, to own them in publick as well as in private.

And indeed Your affection to these Studies, and the Students of it, and that nimble apprehension which all that know You, must acknowledge in You, enforce me to doe something, by which I may render that due observance which I owe unto You.

Here is indeed but little, that is worthy Your perusal, nay, the whole would worthily deserve to be rejected by You, but that I know Your own goodness, will accept of any thing from me, which will but probably adde to the Talent of others knowledge, though it cannot to Your own; this maketh me confident of a Candide acceptance, of what I have here published for the publick good, from him who taketh the boldness to subscribe himself,

SIR

Yours Humbly Devoted,

to serve You

JOHN NEWTON.

TO THE
READER.

THere being ſo much already written by others and my ſelf concerning the Doctrine of plain and Spherical Triangles, (both for the practical reſolution of all the Caſes by Numbers Natural and Artificial, and alſo Geometrical Demonſtration of thoſe Problems, upon which the ſeveral proportions doe depend;) This three-fold Tract with which I preſent thee now, may ſeem to be uſeleſs and in vaine; and therefore it will be at leaſt convenient, to tell thee why, as well as what, I have written; Now the onely reaſon for which I at firſt Collected theſe plain Principles of Geometry was indeed for their ſakes who have little or no Arithmetical skill, yet being done, I hope their uſe will be of a larger extent, and ſerve as an Introduction to the Arithmetical part, and doubtleſs it is ſome ſatisfaction to ſee the agreement, that is between the Mechanical and the Arithmetical reſolution of Problems.

In the firſt part of this Treatiſe: three Sorts or Species of continued Quantity, Lines, Plains *and* Solids *are briefly and plainly defined, ſome*

principal affections proper to each of these severally, and in reference to one another joyntly are explain'd, the Mechanical solution of plain Triangles, by several sorts of lines, as Sines, Tangents, Secants, Chords and equal parts, *is fully shewed, with sundry Problems, for the describing of all sorts of* Right lined Planes; *as for the finding of the Superficial contents of those plains, and of plains that are circular, with the solidity of all regular Bodies, we have for the most part made use of* Logarithms; *and must therefore intreat the* Reader *to inform himself of the manner of working by them, either by what I have written of them in* Trigonometria Britannica, *or by some other Tract of the like nature.*

In the second Part of this Treatise, thou hast a very short, yet plain Description of the Cœlestial and Terrestrial Globes, *with there Use; first, in* Spherical Triangles *in the General, and then in some particular* Astronomical *and* Geographical Triangles *in which we have shewed the number of Problems contcined in every particular Triangle, by which means, thou mayst readily find the Triangle to which the things given doe belong, as well as the resolution of the Problem it self.*

In the third Part of this Treatise, I have gone over

over the work of the Globe again, in which by that little which is their said of Spherical Triangles, *any of the Problems mentioned in the use of the Globe, may be projected and resolved, or any other of the like nature, some other uses of this projection are there explain'd, in reference to* Astrological *and* Gnomonical *Problems, of Gnomonicks, I have been the more large, because the projection of the Sphere upon the plain of most great circles, is thereby cleered. The Demonstration of the Rules by which the circles of the Globe may be thus projected, hath a dependance upon so many Problems, that if I had inserted it here, it must have been either clouded by reason of the many references unto* Euclide, Theodosius, *and* Apollonius; *or this Treatise would have swelled beyond the bulk at first intended, and the limits now prescribed me, and therefore I have chosen rather to omit it; leaving what here is written to be perused, & used by thee, as thou findest occasion for it.*

J. NEWTON.

THe several sorts of lines whose description and use is shewed in this Treatise, are at one view presented to thee in the following Scale, not that I would confine thee to this form of placing the lines, but shew thee onely what lines may be necessary (at least convenient) for thy use: in this, or any other form, more suitable to thine own Genious, thou mayest be furnished with these lines upon a Scale, or with any other Mathematical Instrument in Brass or Wood, by Mr. *Anthony Thompson* in *Hosier-lane*.

TO THE READER.

over the work of the Globe again, in which by that little which is their said of Spherical Trian-gles, any of the Problems mentioned in the use of the Globe, may be projected and resolved, or any other of the like nature, some other uses of this projection are [illegible] explained, in reference [illegible] Astronomical and [illegible] [illegible] I have [illegible] [illegible], being the projection of the Sphere upon the plane of any great circle [illegible] [illegible] [illegible] [illegible] [illegible] by which the [illegible] [illegible] may be [illegible] projected, hath a dependance upon so many Pro-blems, that if I had [illegible] it here, it must have [illegible] [illegible] [illegible] [illegible] [illegible] many [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] Treatise would have swelled beyond the [illegible] first intended, and the limits now [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] used by those, that [illegible] [illegible] [illegible]

J. [illegible]

[illegible] severall [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] this Treatise [illegible] some view [illegible] [illegible] [illegible] scale, not that I would [illegible] [illegible] [illegible] [illegible] the lines, but shew the manner which [illegible] may be need-[illegible] [illegible] deft convenient) for the [illegible] [illegible] [illegible] [illegible] more suitable to their own [illegible] [illegible] [illegible] [illegible] with their Instruments, [illegible] or with any other [illegible] [illegible] Instrument in Brass or Wood, by [illegible] [illegible] in [illegible].

PRACTICAL Geometry,

CHAP. I.

Of the Definition and Division of Geometry.

GEOMETRY is the art of meaſuring well.

2 To meaſure well, is to declare the nature, force, property, proportion and uſe, of any thing that is meaſurable.

3 The Subject of Geometry, is Magnitude or continued quantity, whoſe parts are conjoyned by a common term, or limit. And a term is the extream of a Magnitude.

4 Magnitude is either a Line, or ſomething made of a line, or lines.

5 A Line is a magnitude conſiſting onely of length, without either breadth, or depth, the term, or limit whereof is a Point. For every line is made, continued, and bounded, with a point.

6 A point therefore is no quantity, but the begining of all continued quantities, which are divisible in power infinitely: from the imaginary motion whereof is produced a line: as if the point A be imagined to move from A to B, this point shall by its motion, trace out the line A B.

Fig. 1.

7 A line is either considered Simply by it self, or else comparatively with another line. A Line considered Simply of it self, is either Right, or Oblique.

8 A Right line is that which lieth equally between his points. As the line A B lieth straight and equally between the points A and B, without any going up, or coming down on either side.

9 An Oblique line is either circular, or mixt. A Periphery, or Circular line, is that which is equally distant from the middle of the comprehended space, which middle is called the Center, and the distance between that Center and the circumference, is called the Radius.

All other lines besides the Right and Circular, may be called mixt, or compounded lines, of which there is little use in Geometry.

10 Hitherto of lines considered Simply by themselves, lines compared to one another, are either of the same, or of different Species.

11 Lines compared together of the same Species: are either parallel, or angular.

12 Parallel lines are such as are equidistant in all places, and are either right lined, or circular. Right lined parallels are such which being in one and the self-same plane, and produced infinitely on both sides, do never meet in any part. As the lines A B and H G, but Circular parallels are such Circles, or arches, as are drawn upon the same center.

Right lined parallels may be drawn by the Proposition following.

P R O-

PROPOSITION 1.

To draw a right line parallel to a right line given, according to any distance assigned.

LEt A B be a right line given, to which a parallel is to be drawn at the distance of C D. In the line A B let there be two points taken a little distant one from another, as E and F, from which at the distance of C D given, let there be described two arches, a right line as H G, drawn by those two arches shall be parallel to the line A B given. *Fig.* 1.

13 Angular lines are such as inclining or bowing to one another, touch one another, but not in a direct line. As the right lines H E, and F G, meeting in the point E, do make the right lined angle H E F. *Fig.* 2.

14 An angle is either Right or Oblique.

15 A Right angle is that, whose legs or sides are perpendicular to one another, that is, whose legs stand so, that one of them doth not incline to the other any way, but maketh the comprehended space one both sides equal. Thus the line A B is perpendicular to the line C D, and the angles A B C and A B D are right, and equal to one another, because the spaces between A B and D B, and between the lines AB and B C are equal.

PROPOSITION 2.

From a point assigned without a right line given, to let fall a perpendicular.

WHen the point assigned is neer the line given, as the point C is to the line A B, from the center C, with any distance of your compasses, *Fig.* 3.

intersect the line A B in the points A and B, from which points the compasses being opened to a greater extent then the former, describe two arches above and beneath the line A B, which may cut one the other in the points H and G, through which intersections a right line being drawn shall necessarily pass through the point C, and be perpendicular to the right line A B.

When the point assigned shall be at a greater distance from the line given, doe thus. Through the

Fig. 4. point C draw the right line E C at any acute angle with the line A B, and let the line E C be bisected in the point D, from whence at the distance of D C or E D let the line A B be cut in F. Then shall the right line C F be perpendicular to A B as was required.

Fig. 5. *Another way.*

From any point as at D neer to the end of the line A B, at the distance of C D, describe two arches above and beneath the line A B, and then from the point E, taken at some distance from the point D, describe two other arches at the distance of E C, cutting the former in C and F, then shall the right line C F be the perpendicular sought.

16 An Oblique angle is that whose legs or sides do incline to one another upon this side more then upon that.

17 An Oblique angle is either Obtuse or Acute.

18 An Obtuse oblique angle, is that which is grea-

Fig. 2. ter than a right, whose legs do so decline, that the space comprehended by them, is greater than in a right angle, as the angle B E D is greater than the right angle C E D.

19 An acute oblique angle, is that which is less than a right, whose legs or sides do so incline, that the space comprehended by them, is less than in a right

angle

angle, as the angle A E B is less than the right angle A E C.

20 The measure of an angle is the arch of a Circle described upon the angular point, and intersected between the sides of the angle sufficiently prolonged. As the arch B C described from the angle and center E, is the measure of the angle B E C: but of this measure there can be no certain knowledge, unless the quantity of that arch be expressed in numbers. Fig. 6.

21 Every circle therefore is supposed to be divided into 360 equal parts, called Degrees, and every Degree into 60 minutes, every minute into 60 seconds, and so forward. This Division of the circle into 360 parts we shall retain, but every degree we will suppose to be divided into 10 parts, and every one of those into 10 more, and so forward as far as you please. And thus all calculations will be much easier, and no less certain.

22 A Semicircle is the half of a whole circle conteining 180 degrees.

23 A Quadrant, or fourth part of a circle is 90 deg. And seeing that a right line falling perpendicularly upon a right line, doth make the angles on both sides equal, and cutteth a Semicircle into two equal parts, the fourth part of a circle or 90 degrees must needs be the measure of a right angle.

24 A Right line falling upon a right line, maketh either two right angles, or two angles equal unto two right. Thus the right line B E falling upon the right line A D, maketh the angles A E B & B E D together, equal unto two right angles. For the Semicircle A B C D is the measure of them both: if either of them therefore be known, the other is also known.

25 When two angles are made by the meeting of two right lines, one of them is the complement of the other to a Semicircle, which doth contein two right angles, or 180 degrees: therefore if the angle

given shall be subtracted from two right angles, what remaineth shall be the other angle.

Fig. 6. EXAMPLE.

A Semicircle, or two right angles contein	180 *deg.*
From which subtract the angle AEB	64
The remainer is the angle BED	116

26 Complements of arches or angles are so called, with reference either to a Quadrant or a Semicircle; if therefore a right angle be cut into two equal or unequal parts, either part being known, the other also is known; for if the known part be subtracted from the fourth part of a circle or 90 degrees, the remainder shall be the other part.

EXAMPLE.

A Quadrant or right angle is	90 *deg.*
From which subtract the angle AEB	64
The remainer is the angle BEC	26

PROPOSITION 3.

To a point assigned upon a right line given, to make a right lined angle, equal to an angle given.

Fig. 7. LEt C be the point assigned in the given line AH, upon which it is required to make an angle, equal to the angle EDF: From the point D as a center, at any extent of the Compasses, describe the arch EF, between the sides of the angle given: And with the same extent describe the arch HK, from the point C, which by your Compasses you may make equal to the arch EF; then draw the right line CK, so shall the angle HCK be equal to the angle EDF given; because they are subtended by equal arches of equal circles.

CHAP.

CHAP. II.

Of the dividing of right lines into any number of parts, and according to any proportion aſsigned.

HItherto we have treated of right lines, as they are either parallel, or angular, we will now ſhew how any right line may be divided into any number of equal parts, and according to any proportion given; for which purpoſe we will here premiſe theſe three Axioms, upon which ſome of the Propoſitions following do depend.

AXIOM 1.

If any right lines ſhall be interſected by many parallel lines, the interſegments ſhall be proportional.

LEt the two right lines A D and A E be interſected with the parallels F G, H I, K L, and M N: I ſay the interſegmenes A B and A C, as alſo B E and C D are proportional; that is, if A B be the half of A E, A C ſhall be alſo the half of A D; and if A B ſhall be a third part of A E, A C ſhall be alſo a third part of A D, and ſo of any unequal proportion; The reaſon is plain from the figure, in which H I doth cut off a third part of the whole ſpace F G N M; and therefore by conſequence it cutteth off a third part of every line, which is or may be drawn through that ſpace. *Fig.* 8.

Hence

Hence it followeth,

AB . AE :: AC . AD.

Also by inversion,

AE . AB :: AD . AC : *Or thus*, AB . AC :: AE . AD.

AXIOM 2.

If four right lines be proportional, the right angled figure made of the two means, is equal to the right angled figure made of the two extreams.

Fig. 9. LEt the four proportional lines be AB two foot, EF three foot, FG six foot, and BC nine foot: I say then, that the right angled figure made of the two mean lines, EF and FG, that is, the right angled figure EFGH is equal to the right angled figure of the extreams AB and BC; that is, to the right angled figure ABCD; for as twice 9 is 18, so likewise three times 6 is 18.

1 Consectary

If four right lines be proportional, three of them being given, the fourth is also given; for the rectangle of the two means being divided by either of the extreams, the quotient will give the other extream.

Example.

Let the first extream be given 2, with the two means 3 and 6, the rectangle or product of 3 and 6 is 18, which being divided by 2, the quotient is 9, the other extream.

2 Consectary.

If three right lines be proportional, that is, if the first be to the second, as the second to the third, the square of the

mean

mean or second shall be equal to the rectangle made of the extreames; because the middle term is put twice, in this manner.

As 2. *is to* 4 :: *So is* 4. *to* 8.

And so it is all one as if there were four proportionals: what was therefore said of four proportionals, is also to be understood of three.

AXIOM 3.

If the sides of a right lined angle be intersected with parallels, those parallels shall be proportional to the conterminate segments.

If the sides of the angle D A E be intersected with the parallels D E and B C, As the parallel D E, is to the parallel B C, so is the segment of the side D A, to the other segment of the same side B A. *Fig.* 10.

For Demonstration sake. Through the point B, draw the right line B F parallel to the right line A E, which shall cut D E and D A proportionally, in the points F and B, *By the* 1 *Axiom of this Chapt.* and therefore: D A . B A :: D E . F E, or B C, for that F E and B C are equal.

In like manner, the right line C F being drawn through the point C parallel to A D, the right lines E A and E D shall be cut proportionally in the points F and C, and it shall be,

E A . C A :: E D . E F. or B C, for that E F and B C are equal.

And because it is D A . B A :: D E . B C.
And also E A . C A :: D E . B C.

Therefore D E . B C $\left\{ \begin{array}{l} :: D A . B A \\ :: E A . C A \end{array} \right.$

 These

These Axioms being premised, we come now to the Propositions themselves, by which a right line may be divided into any number of parts, and according to any proportion assigned.

PROPOSITION 1.

× *To divide a right line given into any number of equal parts.*

Fig. 11. LEt it be required to divide the right line A B, into 9 equal parts; from the extream points of the given line A and B, let there be drawn two parallel lines, then from the point A at any distance of the compasses, set off as many equal parts wanting one, as the given line is to be divided into, which in our Example is 8, and are noted with the letters E. F. G. H. K. L. M. N. and from the point B set off the like parts in the line B C, and let them be noted with O P. Q R. S. T. V. X. then shall the parallel lines E X. F V. G T. H S. K R. L Q. M P. and N O, divide the right line A B into 9 equal parts as was required.

And thus may that line be made, which is commonly called a Diagonal, and taketh up one side of this Scale or Ruler.

PROPOSITION 2.

Two right lines being given, to find a third proportional.

Fig. 12. LEt a third proportional be required to the two right right lines given A B and A C; make A C perpendicular to A B, at the point A, and let A B be continued at pleasure, and draw the line B C, lastly, from the point C, erect the perpendicular C D, and where that cutteth the line B A produced, make a mark, as suppose at D, so shall A D be the third proportional required, for B A . A C : : A C . A D.

Otherwise

3.

4.

15.

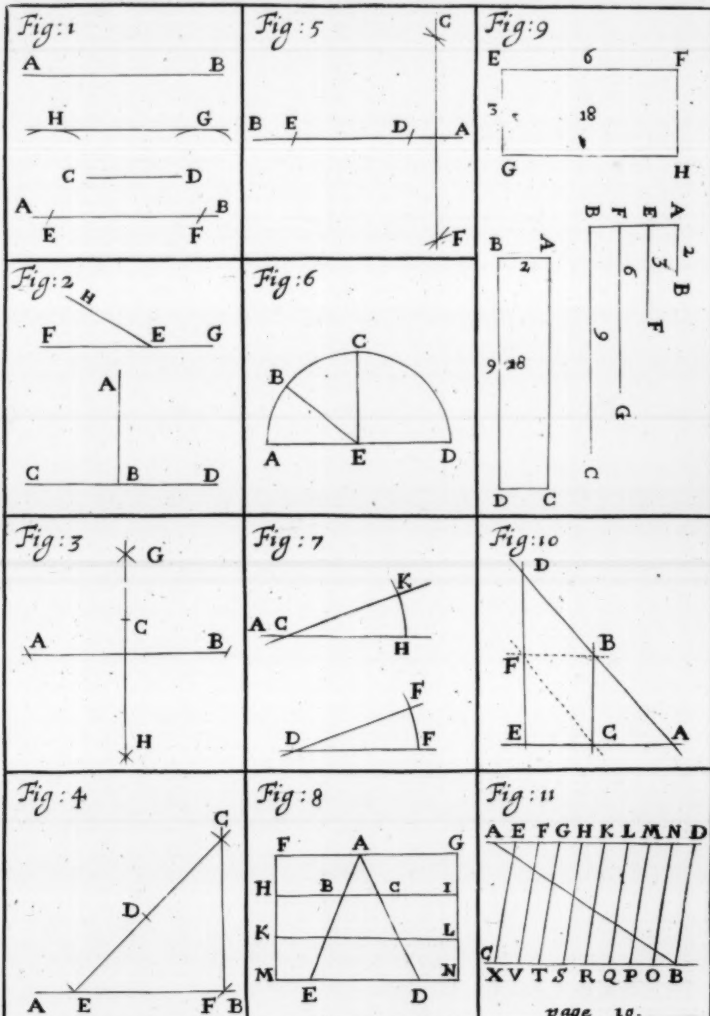
Fig:1
A B
H G
C D
A B
E F
Fig:2
H
F E G
A
C B D
Fig:3
G
C
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Fig:4
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A E F B
Fig:5
C
B E D A
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Fig:6
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K
A C
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Fig:8
F A G
H B C I
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M N
E D
Fig:9
E 6 F
3 18
G H
B A
2
9 18
D C
B F E A
2
3
6
9
B
F
G
C
Fig:10
D
B
F
E C A
Fig:11
A E F G H K L M N D
C
X V T S R Q P O B
page 10.

Otherwise.

Let the right lines given A B and A C, be joyned together at any acute angle, and A C and A B being continued make B E equal to A C, then draw B C, and D E parallel thereunto, so shall D C be the third proportional sought; for, A B . B E = A C : : A C . C D. Fig. 13.

PROPOSITION 3.

Three right lines being given, to find a fourth proportional.

LEt the three given lines be A B, B C and A E, to which a fourth proportional is required: draw A D at any acute angle to A B in the point A, and make E D parallel to C B, so shall A D be the fourth proportional. For, A B . B C : : A E . E D. Fig. 13.

PROPOSITION 4.

Two right lines being given, to find a mean proportional between them.

LEt the two right lines given be A B & B C, which let be made into one line as A C, from the point D being the middle of the line A C, as from a center, describe the arch C L A, to which semicircle let C A be the Diameter, then from the point B, erect the perpendicular B L cutting the circumference in L, so shall B L be the mean proportional required; for, C B . B L : : B L . B A. Fig. 14.

PROPOSITION 5.

To cut a right line given into extream and mean proportion.

A Right line is said to be divided into extream and mean proportion, when the whole is to the greater Fig. 15.

Fig.15. part, is as the greater is to the less. Let then the right line B A be given to be thus divided, from the point B erect the perpendicular B F, and draw the right line F A, which let be bisected in D, the other foot of the compasses resting in F: at the distance F B, make F D equal thereunto, and A E equal to A D, so shall the right line A B be divided, as was required; for, A B . A E : : A E . E B.

CHAP. III.

Of Right lines applied to a Circle.

Hitherto we have spoken of right lines as they are considered of themselves; or are compared with other lines of the same species: we will now treat of them in reference to other lines of a different species, that is, to oblique or circular lines.

2 And right lines as they have reference to, or are compared with the circumference of a circle, are either such as are inscribed within it, or applied to it.

3 A right line inscribed in a circle, passeth through or toucheth the Center, as the Diameter and Radius, or is drawn besides the center, as Chords and Sines.

Fig.16. 4 A Diameter is a right line inscribed through the center of the circle, dividing the circle into two equal parts. As the right line G D drawn through the center B, and at both ends bounded with the circumference: The half whereof G B or B D is the Radius of that circumference or circle; For

5 Radius is the half of a Diameter, or a right line drawn from the center to the circumference.

6 A Chord or subtense is an inscribed right line drawn besides the center, that is a right line which being at both ends bounded with the circumference, doth

doth not passe through the center thereof: as the right line C K subtendeth the arch C D K. The half whereof C A which is perpendicular to the Radius, from the term of the same arch, is the right sine of C D half of the said arch C D K; and also the sine of C E G the complement of C D to a semicircle. Now sines are either Right or Versed. *Fig.16.*

7 A Right sine is half of the subtense of the double arch, or a right line which falleth from the term of an arch perpendicularly to the Radius or Diameter.

8 A Versed sine, is a part of the Diameter lying between the right sine and the circumference: As the part of the Diameter A D is the versed sine of the arch C D, and the other part A G is the versed sine of the arch C G.

9 A right line applied to a circle, is either a Tangent or a Secant.

10 A Tangent is a right line without the circle drawn perpendicular to the end of the Radius, and continued to the Secant.

11 A Secant is a right line, drawn from the center of the circle through the term of an arch, and continued to the Tangent: As the right line F D which is perpendicular to the Radius without the circle, is the tangent of the arch C D, or of the arch C G the complement of C D to a semicircle. And the right line B F, which from the center B is continued through the point C in the circumference to the tangent, is the secant of the same arches C D and C G.

12 By these lines thus inscribed or applied to a circle; A scale or ruler may be made to a certain Radius or diameter of a circle assumed, which shall contein, the Chords, Sines, Tangents, and Secants of every degree in the Quadrant, by help whereof the several Cases both of Plain and Sphærical Triangles may be easily resolved.

13 Here therefore we will shew how a circle may be divided into 360 degrees or parts, or a Semicircle

 into

into 180 equal parts or degrees; which being done, the lines of Chords, Sines, Tangents & Secants to every degree in the Quadrant, may be drawn without much trouble.

The manner by which a circle may be divided into its parts, is thus.

Fig.17. 1 THrough the center A draw the diameter B C, upon which setting one foot of the compasses in A, the other being extended to B or C, describe the semicircle B G C.

2 Setting one foot of the compasses in B at any extent of the compasses greater than B A, describe an occult arch, and at the same extent of the compasses, setting one foot in C, describe another occult arch, so as it may cut the former, and from the point where those arches doe intersect one another, draw a line to the center A, so shall the semicircle B G C be bisected into two equal parts, or Quadrants, each Quadrant conteining 90 degrees.

3 Because the Radius of a circle, is equal to the Chord of 60 degrees in the same circle, if you set the Radius A B or A C, from G downwards towards C, it will reach to H, and the arch G H shall be 60 degrees, and the same extent of the compasses being set from C upwards, will reach to D, and the arch C D shall be 60 degrees also, and the Quadrant G C shall be divided into 3 equal parts, each part conteining 30 degrees.

4 Because if the Radius of a circle be cut into extream and mean proportion, the greater segment shall be the side of a Decangle or Chord of 36 degrees, therefore the Radius A B or A C being so cut, *by the 5th. Propos. of the 2 chap.* the greater segment B E, shall in this circle be the Chord of 36 degrees, which being set from C upwards, will reach to F, and the arch F H

is

is the chord of 6 degrees, by help whereof the Quadrant G C may be distributed into 15 parts, each part conteining 6 degrees, and each of these may be subdivided, first into two, and then each of those subdivisions into three more, so will the Quadrant be divided into 90 equal parts or degrees: and in like manner may the Quadrant B G be divided also: at every tenth degree there is a little line drawn from the center, and numeral figures set to each line, the better to number the degrees. Fig. 17.

14 From a Semicircle thus prepared, the lines of Chords, Sines, Tangents, and Secants, may easily be transferred upon a Ruler.

For the extent from B to G, set from K to L, is the chord of 90, and from B to M, will set off K M 80, and so the rest, and the line K L shall be the line of chords.

15 Sines (as hath been said) are either Right, or Versed, lines drawn parallel to the diameter B C at every tenth degree, shall cut the Radius or whole sine A G into 9 unequal parts, and the line A G shall be the line of Right Sines.

16 But the lines falling perpendicularly upon the Diameter B C from every tenth degree, as the lines Q Y, P X, &c. are the right sines of 40 and 50 degrees, equal to those in the line A G, and doe also, cut the diameter B C into 18 unequal parts, and the diameter B C shall be the line of Versed Sines.

17 Lines drawn from the center A through every tenth degree of the circle, to the right line B Z, perpendicular to the Radius A B, shall cut the said perpendicular line into unequal parts, and the line B Z shall be the line of Tangents.

And those very lines which being drawn from the center A, to the perpendicular B Z, shall be the Secants of those arches, through which they are drawn, numbered from the point C, which being transferred to the line A G, the said line A G shall be the line of Secants.

PRO-

PROPOSITION 6.

The arch of a circle being given, to describe the whole periphery thereof.

Fig. 46. LEt A B C be an arch given, whose periphery may easily be found in this manner: Let there be three points taken in the given arch, where you please, as A, B, and C, one foot of your compasses being placed in the point A, opening the other to more than half the distance of A B, describe the arch of a circle: then the compasses remaining at the same distance, seting one foot in B, describe another arch so as it may cut the former in the two points G and H, through which draw the right line H G towards that part in which you suppose the center of the circle will be.

In like manner, with the same or any other, greater or lesser extent of the compasses, let there be drawn two other arches from the points B and C, cutting each other in E and F, and draw the line E F, where this line shall intersect the former, which here is at the point D, there shall be the center of the circle to be drawn.

Consectary

Hence therefore it followeth: That any three points as A, B, and C, being given, not lying in a right line, may be brought into a Circle.

CHAP. IV.

Of Right lined Triangles.

HItherto we have spoken of the first kind of Magnitude, that is, of Lines; as they are considered of themselves, or amongst themselves.

2 The second kind of Magnitude, is that which is made of lines, that is, a Figure.

3 A Figure, is that which is comprehended by one or more limits, yet every quantity which hath bounds or limits, is not a figure, and therefore we doe not say, that a finite line, or an angle which is comprehended by the legs or sides thereof, is, or may be called, a figure: but that quantity which hath a bound or limit on every side. Whether it be bounded with one limit, as a Circle, a Sphere, a Spheroide, and such like; or with more, as a Triangle, a Quadrangle, a Pyramis, a Cube, &c.

4 The terms or limits of every figure, are either lines, or super ficies.

5 A figure which is terminated by lines is a superficies.

6 A Figure which is bounded or limited with superficies, is a body or solid.

7 A Superficies is a magnitude both long and broad, long by reason of the length of the line, and broad by reason of the motion of the line, which it maketh cross-wise.

8 A Superficies is either right lined, curvilined, or composed of both.

9 A Right lined plane or superficies, is that which is terminated with right lines. It is either a Triangle or a Triangulate.

10 A Triangle or the first right lined figure, is that which is comprehended by three right lines. It is distinguished either from the sides, or from the angles.

11 In respect of the sides, a Triangle is either *Isopleuron*, *Isosceles*, or *Scalenum*.

An *Isopleuron* Triangle, is that which hath three equal sides.

An *Isosceles*, which hath two equal sides.

A Scalenum, whose three sides are all unequal.

12 In respect of the angles. A Triangle is either Right, Obtuse or acute.

A Right angled Triangle, is that which hath one right angle and two acute.

An Obtuſe angled Triangle, hath alſo two acute angles, but one obtuſe.

An acute angled Triangle hath all the three angles acute.

PROPOSITION 1.

Upon a right line given, to make an ordinate triangle, that is, an equiangled and equilateral triangle.

Fig.18. LEt A E be a right line given, upon which it is required to make an equiangled and equilateral Triangle; upon the points A and E as upon two centers, deſcribe two arches, either above or beneath the line, cutting each other in F, from whence let there be drawn the right lines F A and F E: ſo ſhall the Triangle A F E be equiangled, and equilateral.

PROPOSITION 2.

Upon a right line given, to make an Iſoſceles triangle.

LEt B A be the right line given, from the points A and B as from two centers, but at a leſſer extent of the compaſſes then A B, if you would have A B to be the greateſt, at a greater if you would have it to be leaſt, deſcribe two arches, cutting one another in F: from whence draw the lines F A and F B, ſo ſhall the triangle A F B have the two ſides A F & F B, equal by reaſon of the equal extent of the compaſſes, & either of them greater or leſſe then A B, as was required.

PROPOSITION 3

To make a triangle whose three sides, shall be equal to any three lines given, if every two lines be more then the third.

LEt A. B. C, be the three lines given, any two of which are more then the third, (for otherwise a Triangle could not be made of them.) *Fig.* 19.

Make D E equal to one of the given lines, suppose B, and from D as a center, at the extent of A, describe the arch of a circle: in like manner, from E at the distance of C, describe another arch intersecting the former in F, then shall the right lines F E, F D, and D E comprehend a Triangle, whose three sides shall be equal to the three right lines given A. B. and C, as was required.

CHAP. V.

Of Quadrangles.

HItherto we have spoken of the first kind of right lined plaines, that is, of Triangles.

The second sort of right lined plaines, is called a *Triangulate.*

1 And a *Triangulate* is a plain composed of Triangles.

2 The sides of a *Triangulate*, are in number more by two, then the Triangles of which it is composed. Because two right lines cannot on every side include a figure, but that three at the least are required, which doe constitute a Triangle, or first right lined figure: besides a Triangle is a figure composed of Triangles, and may be resolved into them: which resolution into

Triangles is done beſt, by drawing right lines from any one angle to all the reſt: yet ſo, as that they may fall upon the oppoſite angles; for that right lines cannot be drawn to the two neareſt angles, for ſo they would be the ſame with the ſides of the *Triangulate*: Fig. 20. as in the Figure A B C D E F, if right lines be drawn from F to A and E, they will be the ſame with the lines F A and F E, by drawing whereof, there would be no angle made, and therefore alſo no Triangles: but by drawing the right lines F D. F C. & F B, in the ſix-ſided Triangulate A B C D E F, we have the four Triangles, F A B. F B C F C D. and F D E.

Every *Triangulate* therefore, may be reſolved into as many Triangles as there are angles in it, thoſe two onely excepted to which right lines are not drawn. And therefore a *Quadrangle* which hath four ſides and four angles, can be reſolved but into two Triangles: a *Quincangle* into three, a *Sexangle* into four and ſo forward, the Triangles being ſtill in number fewer by two, then there are ſides in the *Triangulate* given.

3 A *Triangulate*, is either a *Quadrangle*, or a *Multangle*.

4 A *Quadrangle*, is a plane comprehended by four right lines, and is either a *Parallelogram*, or a *Trapezium*.

5 A *Parallelogram*, is a *Quadangle* whoſe oppoſite ſides, are parallel or equidiſtant, and is either Right-angled or Oblique.

6 A Right-angled *Parallelogram*, is that which hath every angle right, and is either a Square or an Oblong.

7 A Square, is a Right-angled *Parallelogram*, whoſe four ſides are equal, and the angles Right.

8 An Oblong, is a parallelogram whoſe angles are all right, but the ſides unequal.

9 An Oblique angled *Parallelogram*, is that whoſe angles are all oblique; and is either a *Rhombus*, or a *Rhomboides*.

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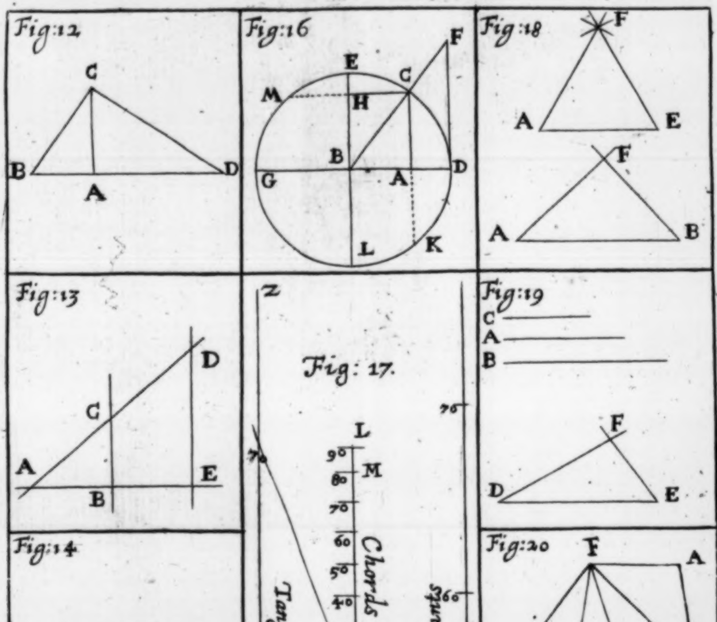
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Fig:20
Fig:15
Tangents
Chords
Secants
Sines
Versed Sines

10 *A Rhombus*, is an oblique angled parallelogram of equal ſides.

11 *A Rhomboides*, is an oblique angled parallelogram of unequal ſides.

12 *A Trapezium*, is a quadrangle, but not a parallelogram, and it is either right angled or oblique.

13 *A* Right angled *Trapezium*, hath two oppoſite ſides parallel, but unequal, and the ſide between them perpendicular.

14 An oblique angled T*rapezium*, is a Quadrangle, but not a parallelogram, having at leaſt two angles thereof oblique, and none of the ſides between the parallels perpendicular.

PROPOSITION 1.

Upon a right line given, to deſcribe a Square.

UPon the given line A B, erect the perpendicular A D, equal to A B, and upon the points B and D at the diſtance of A B, deſcribe two arches interſecting one another at E, and draw the lines B E and D E, then ſhall the right angled figure A E, be a ſquare as was required. *Fig.* 21

PROPOSITION 2.

To make an oblong of two lines given.

LEt the two given lines be A B and C D, make E F equal to C D and E G equal to A B, and perpendicular unto E F: then at the diſtance of E G equal to A B upon the point F, deſcribe an arch, and at the diſtance of E F equal to C D upon the point G, deſcribe another arch interſecting the former in H: Laſtly, draw the lines H G and H F, ſo ſhall the right angled figure E H be oblong, as was required. *Fig.* 22.

PROPOSITION 3.

Upon a given line to describe a Rhombus, which shall have an angle equal to an angle given.

Fig. 23. UPon the given line A B, make the angle D A B equal to C, and A D equal to A B, and at the extent of A B, upon the points B and D, describe two arches intersecting one another in E, and draw the lines E B and E D, then shall A E be a *Rhombus*, as was required.

PROPOSITION 4.

Of two lines given to make a Rhomboides, having an angle equal to an angle given.

Fig. 24. LEt the given lines be A B and C D, make E F equal to C D and E G, equal unto A B, and the angle G E F equal to C, and upon the points G and F, describe two arches intersecting one another in H, then draw the lines F H & G H, so shall E H be the *Rhomboides*, which was required.

CHAP. VI.

Of Multangles.

HItherto of Quadrangles, or the first sort of Triangulates: The second we call *Multangles*.

1 A Right lined *Multangled* plain, is that, which is comprehended by more than four right lines: And by this general name, *Euclide* doth comprehend all other right lined figures: As the *Quincangle*,

Quincangle, *Sexangle*, *Septangle*, and ſuch like. Thus alſo from the number of the ſides comprehending the figure, it may fitly enough be termed, a five-ſided, ſix-ſided, ſeven-ſided figure.

2 A *Multangle* right lined plain or Polygon, is either ordinate and regular, or inordinate and irregular.

3 *Ordinate* and regular Polygons, are ſuch, which are conteined by equal ſides and angles: as a *Pentagon*, *Hexagon*, and ſuch like.

4 Inordinate or irregular Polygons, are ſuch as are conteined by unequal ſides and angles.

PROPOSITION I.

A Multangle or right lined Polygon being given, whether regular or irregular, to make another right lined Polygon, upon a right line given, like to it, and in like manner ſituated.

UPon the right line A L, let it be required to make a right lined ſixangled figure, like & in like manner ſituated as the figure C D E F G H. The ſides C D and C H being produced, from the angle C, draw right lines to all the other angles: as C E. C F. C G, and upon C H make C K equal to A L, and K P parallel to H G, and P M parallel to G F, and ſo the reſt till all the ſides, except C H and C D, ſhall have there parallels, ſo ſhall be made what was required: that is, the ſixangled figure C K P M N O ſhall be like to the ſixangled figure C D E F G H, and in like manner ſituated, and that upon the right line C K, equal to A L the right line given. *Fig.* 25.

PRO-

PROPOSITION 2.

To make a regular Pentagon and Decagon in a given Circle.

Fig.26. LEt there be a ſemicircle deſcribed upon the center A, whoſe diameter is B C, from the center A, erect the Radius A D perpendicular to the diamater, and let the Semidiameter A C be cut in the middle, ſuppoſe at E, and the diſtance E D ſet upon the diameter from E to G, and draw the line G D, which ſhall be the ſide of a Pentagon, and A G the ſide of a Decagon to be inſcribed in the circle given.

The right line DG being therefore applied five times in the circles circumference, *viz.* in the points D. I. K. L. and M, the right lines D I. IK. K L. L M. and M D, ſhall make a regular pentagon as was deſired.

In like manner, if the diſtance A G were applied in the ſame circle ten times, the lines drawn to each point will conſtitute a regular Decagon.

☞ *Here we may note*, That a Polygon being inſcribed in a circle, another polygon may be inſcribed in the ſame circle, whoſe ſides and angles ſhall be twice as many as in the polygon given, for every arch being divided into two parts, the right lines or ſubtenſes of thoſe diviſions ſhall be the ſides of ſuch a polygon. A Decagon therefore being inſcribed, an Iſoſagon or figure of twenty ſides may be inſcribed alſo, and a Hexagon being inſcribed, a Dodecagon, or figure of twelve-ſides may eaſily be made.

PROPOSITION 3.

In a circle given to deſcribe a regular Hexagon.

THe ſide of a Hexagon is equal to the Radius of a circle, the Radius of a circle therefore being ſix times applied to the circumference of it, ſhall give you

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ſix points, to which lines being drawn from point to point, ſhall conſtitute the regular *Hexagon*, as was deſired.

PROPOSITION 4.

In a circle given, to make any regular Poligon whatſoever.

DIvide the circumference of the circle, that is 360 degrees, by the number of ſides or angles in the poligon deſired, the number of degrees in the quotient, take from a line the chords ſuteable to the Radius of the circle given, the chord of that arch ſhall be the ſide of the poligon deſired: As if an Octogon were required, the number of degrees in a circle, *viz.* 360, being divided by 8, the quotient is 45: and therefore the chord of 45 degr. ſhall be the ſide of an Octogon: and ſo of any other.

POPOSITION 5.

Upon a right line given to make any regular polygon whatſoever.

UPon the right line *A* B, let it be required to deſcribe a Quincangle, upon the center *A*, at the diſtance of A B, deſcribe the circumference of a circle, and let *A* B be continued from A to C, ſo as B C may be the diameter: and the degrees in a circle, *viz.* 360, being divided by the number of ſides, or angles in the polygon deſired, the degrees in the quotient number from C towards D. Thus for the ſide of a Quinincangle, I number from C to D 72 degrees, becauſe they will be the number of degrees in the quotient, a whole circle being divided by 5, then draw the right line A D, which together with A B ſhall conſtitute the angle of the pentagon to be deſcribed. Therefore if a circle be deſcribed by the three points given B, A, D, and the right lines D F, F F, and F B in Fig.27.

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PROPOSITION 2.

To make a regular Pentagon and Decagon in a given Circle.

Fig.26. LEt there be a ſemicircle deſcribed upon the center A, whoſe diameter is B C, from the center A, erect the Radius A D perpendicular to the diamater, and let the Semidiameter A C be cut in the middle, ſuppoſe at E, and the diſtance E D ſet upon the diameter from E to G, and draw the line G D, which ſhall be the ſide of a Pentagon, and A G the ſide of a Decagon to be inſcribed in the circle given.

The right line DG being therefore applied five times in the circles circumference, *viz.* in the points D. I. K. L. and M, the right lines D I. IK. K L. L M. and M D, ſhall make a regular pentagon as was deſired.

In like manner, if the diſtance A G were applied in the ſame circle ten times, the lines drawn to each point will conſtitute a regular Decagon.

☞ *Here we may note*, That a Polygon being inſcribed in a circle, another polygon may be inſcribed in the ſame circle, whoſe ſides and angles ſhall be twice as many as in the polygon given, for every arch being divided into two parts, the right lines or ſubtenſes of thoſe diviſions ſhall be the ſides of ſuch a polygon. A Decagon therefore being inſcribed, an Iſoſagon or figure of twenty ſides may be inſcribed alſo, and a Hexagon being inſcribed, a Dodecagon, or figure of twelve-ſides may eaſily be made.

PROPOSITION 3.

In a circle given to deſcribe a regular Hexagon.

THe ſide of a Hexagon is equal to the Radius of a circle, the Radius of a circle therefore being ſix times applied to the circumference of it, ſhall give you

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ſix points, to which lines being drawn from point to point, ſhall conſtitute the regular *Hexagon*, as was deſired.

PROPOSITION 4.

In a circle given, to make any regular Poligon whatſoever.

DIvide the circumference of the circle, that is 360 degrees, by the number of ſides or angles in the poligon deſired, the number of degrees in the quotient, take from a line the chords ſuteable to the Radius of the circle given, the chord of that arch ſhall be the ſide of the poligon deſired: As if an Octogon were required, the number of degrees in a circle, *viz.* 360, being divided by 8, the quotient is 45: and therefore the chord of 45 degr. ſhall be the ſide of an Octogon: and ſo of any other.

POPOSITION 5.

Upon a right line given to make any regular polygon whatſoever.

UPon the right line *A* B, let it be required to deſcribe a Quincangle, upon the center *A*, at the diſtance of A B, deſcribe the circumference of a circle, and let *A* B be continued from A to C, ſo as B C may be the diameter: and the degrees in a circle, *viz.* 360, being divided by the number of ſides, or angles in the polygon deſired, the degrees in the quotient number from C towards D. Thus for the ſide of a Quinincangle, I number from C to D 72 degrees, becauſe they will be the number of degrees in the quotient, a whole circle being divided by 5, then draw the right line A D, which together with A B ſhall conſtitute the angle of the pentagon to be deſcribed. Therefore if a circle be deſcribed by the three points given B, A, D, and the right lines D F, F F, and F B in *Fig.27.*

that circle made equal to A B or A D, there ſhall be made a regular pentagon, upon the right line A B given, as was required.

PROPOSITION 6.

In a circle given, to circumſcribe any regular polygon whatſoever.

Let a regular polygon be inſcribed within the circle given by the fourth hereof, and to the extremities of the ſeveral ſemidiameters, drawn from the center to the angles of the polygon inſcribed, let there be erected perpendiculars: thoſe perpendiculars ſhall conſtitute a regular polygon of as many equal ſides and angles circumſcribing the circle, as are in the polygon inſcribrd.

For Example.

Fig. 28. Let a pentagon be inſcribed in the circle, and from the center A to the ſeveral angles, draw the Semidiameters A B. A C. A D. A E. and A F, to whoſe extremities, if there be drawn the perpendiculars G H. H I. I K. K L. and L G, meeting in the points G. H. I. K. and L, there ſhall be deſcribed a regular pentagon about the circle given.

5 And now having ſhewed what a right lined figure is, and how the ſeveral ſorts of them may be deſcribed, we will in the next place ſhew you, how they may be meaſured, both in reſpect of the lines by which they are bounded, and alſo of there area, or ſuperficial content.

6 And firſt, we ſhall ſhew how the lines and angles of all plain figures, eſpecially Triangles, may be meaſured, as being the firſt and chiefeſt of them, and into which (as hath been ſaid) all other may be reduced.

7 The ſides of all plain Triangles, and other plain figures, are meaſured by the ſcale, or line of equal parts.

8 The

8 The angles are meaſured by the lines of Sines, Tangents, or Secants: but chiefly by the line of Chords; how they are to be meaſured by Sines, Tangents, or Secants, ſhall be ſhewed in the ſeveral Problemes, in which they are proper, in this place it ſhall ſuffice to ſhew how any angle may be meaſured by the line of Chords onely.

PROPOSITION 7.

To find the quantity of any right lined angle given, or to lay down an angle to any quantity or number of degrees propoſed.

1 TO find the quantity of an angle given: Let the quantity of the given angle E B D be required, open your compaſſes in the line of Chords, from the beginning thereof, to 60 degrees, (becauſe the Chord of 60 is equal to the Radius in the ſame circle) and ſetting one foot thereof in the point B, with the other, deſcribe the arch D E, then take in your compaſſes the diſtance between E and D, and applying that extent to the line of Chords, it will ſhew you the number of degres conteined in that angle, which in our example will be found to be 40 deg. *Fig.29.*

Secondly, To make an angle to the quantity or number of any degrees propoſed; draw a line at pleaſure as A B, then open your compaſſes to the number of 60 degrees in your line of Chords as before and ſetting one foot in the point B, with the other deſcribe the arch E D, and from the point B let it be required to make an angle of 40 degrees, open your compaſſes to that extent in the line of Chords, and ſetting one foot in D, with the other, make a mark as at E, and draw the line E B, ſo ſhall the angle A B E contein 40 degrees, as was required.

CHAP. VII.

Of the resolution of plain Triangles.

IN the resolution of plain Triangles, the angles onely being given, the sides cannot be found, but the reason of the sides onely, it is therefore necessary that one of the sides be known.

2 In every plain Triangle, two angles being given, the third is also given; and one angle being given, the sum of the other two is also given, because the three angles together are equal to two right, or 180 deg.

3 In a plain Right angled Triangle, one of the acute angles is the complement of the other to a quadrant or 90 degrees.

4 In a Right angled Triangle, two terms (besides the Right angle) will serve for the finding out of the third; so that one of them be a side. In the resolution therefore of these Triangles, there are three terms ingredient; but in Oblique angled Triangles there are four: indeed the Rules and Axioms by which all triangles are to be resolved, doe still suppose three things to be given for the finding of a fourth, but in Right angled Triangles, it is sufficient to have two, because a third, to wit, the right angle, is alwayes given.

5 All plain Triangles, may mechanically be most readily resolved, with the lines of Chords, and equal parts, but because the resolution of them by the lines of Sines, Tangents, and Secants, will much help a beginner to understand the Canons or Axioms by which they are to be resolved in numbers; we will here set down the Axioms themselves, and shew how all Right and olique angled plain Triangles may mechanically be resolved according to those Axioms also.

6 The Axiomes by which all plain Triangles may be

be resolved are four; one of which is proper to right angled plain Triangles onely; the other three are true in all.

7 In right angled plain Triangles, the side subtending the right angle we call the Hypotenusa, as the side B C subtending the angle B A C, and the sides comprehending the right angle we call the legs, as A B & A C. *Fig.29.*

AXIOM 1.

In right angled plain triangles. As either leg, is to the Radius; So is the other leg, to the tangent of the angle opposite thereunto.

Or so is, the Hypotenusa, to the secant of the angle opposite to the other leg.

AXIOM 2.

In all plain triangels. The sides are proportional to the sines of there opposite angles.

THese two Axioms are sufficient for the finding of the things inquired, in any of the seven Cases of Right angled plain Triangles, and for some of the things inquired in oblique also.

In right angled plain Triangles, If the hypotenusa be one of the things given or inquired, you may use the sine or secant of the degrees in the given, or inquired angle.

But if the hyponusa be neither given nor inquired, you may use the sine or tangent of the degrees in the angle given, and of the angle inquired the tangent onely, as by the following Problems it shall be proved: in which we will resolve the Cases in right angled plain Triangles: first by the Scale of equal parts and the line of Chords onely, and then by the lines

of Sines, Tangents, and Secants, in those Cases which are proper for them.

PROBLEM I.

The angles and one leg given, to find the Hypotenusa and the other leg.

Fig.29. IN any Right angled Triangle, let one of the legs be 512, the lesser angle 36.87 degrees, and the greater 53.13 degrees. Draw a line at pleasure, as A B, and at right angles to the point A, erect the perpendicular A C, and by help of your Scale of equal parts, set off from A to B 512, and upon the point B, *by the 7 Prop. of the last Chap.* lay down an angle of 36.87 deg. and draw the line B E, till it cut the perpendicular A C, then measure the lines B C and A C by the Scale of equal parts, so shall the one, *viz.* B C, be the hypotenusa, and A C the other leg inquired.

This Problem doth contein two of the seven Cases in Right angled plain triangles, resolveable by the former Axioms.

The angles and one leg given, to find	1 The Hypotenusa. 2 The other leg.

In the first of these Cases the hypotenusa is inquired, & therefore in the proportion we may make use either of the line of Sines onely, or onely of the line of Secants, or of the lines of Tangents and Secants joyntly, with the Scale of equal parts.

Fig.30. By the line of Sines and equal parts, this Case may be thus resolved; draw a line at pleasure as A B, and make B D equal to the sine of the angle opposite to the given leg A B 512, *viz.* 53.13 deg. and upon the point D, erect the perpendicular D E, and the perpendicular A C upon the point A, then open your compasses to the Radius of your Scale, and setting one foot in B, turn the other till it tough the line D E, and there make a mark, supoose at E, and draw the line B E, which

which must be continued till it cut the other perpendicular A C, then is B C the hypotenusa as before, and the proportion is,

B D . B A : : B E . C B.

Secondly, By the line of Secants and equal parts, this Case may be thus resolved: Draw A B at pleasure as before, and from A to B set off the leg given, and make B F equal to the Radius of your Sclale, and upon the point F, erect the perpendicular F G, then open your compasses in the line of the Secants, to the quantity of the angle adjacent to the given leg A B, and setting one foot in B, turn the other about, till it touch the perpendicular F G suppose at G, and draw the line B G, which must be extended till it cut the other perpendicular A C erected upon the point A as before, and thus also the right line B C shall be the hypotenusa inquired, and the proportion,

Radius B F . B A : : B G . B C.

Thirdly, By the lines of Tangents and Secants jointly with the Scale of equal parts, this Case may be resolved, draw a line at plesure as A B, and upon the point A erect the perpendicular A C, then number the given leg from A to B, and make B F equal to the Radius of your Scale, and upon the point F, erect the perpendicular F G, and upon your line of Tangents open your compasses to the tangent of the angle adjacent to the given leg A B, and set that distance from F to G, and draw the line B G, which must be extended till it cut the other perpendicular A C as before, so is B C the hypotenusa inquired, and making C H equal to the Radius of your Scale, the perpendicular K H shall be the tangent of the angle at C, and K C the secant of the same angle, and the proportion, *Fig. 31*

K H . B A : : K C . B C.

To find the other leg.

In the second of these Cases the other leg is inquired; and

and therefore in the proportion we may use the line of Sines onely, or onely the line of tangents.

By the line of Sines, the Triangle must be protracted, as hath been already shewed, in the use of the line of Sines to find the hypotenuse, and the proportion,

BD.DE::BA.AC.

By the line of Tangent, the Triangle must be protracted, as hath been shewed in the third way of finding the hypotenuse, and the proportion is,

Radius BF.FG::BA.CA.
Or, KH.HC::BA.CA.

PROBLEM 2.

The Hypotenusa and oblique angles given, to find the legs.

Fig.29. IN any Right angled plain Triangle: Let the given hypotenusa be 640, and one of the angles 36.87 deg. the other is the complement thereof to a quadrant 53 deg. 13 m. Draw a line at pleasure as BA, and upon the point B protract one of the angles given, suppose the lesser, and draw the line BC, and by your Scale of equal parts, number from B to C the hypotenusa given, and from the point C to the line AB, let fall the perpendicular CA, then is BA one, and CA the other of the legs inquired.

This Problem conteineth but one of the seven Cases in a Right angled plain Triadgle, *viz.* the third Case, as we have placed them, and may be resolved as hath been shewed in the preceding Problem, either by the line of Sines onely, or onely by the liue of Secants, or by the lines of Tangeuts and Secants joyntly with the Scale of equal parts, the manner of protraction hath been already shewed, and the proportions are but the inverse of the former.

The

The Proportion by Sines.

BD . BA :: BE . BC. Therefore BE . BC :: BD . BA.

The Proportion by Secants.

BF, BA :: BG . BC, Therefore BG . BC :: BF . BA.

The Proportion by Tangents and Secants.

KH.BA :: KC . BC. Therefore CK , BC :: KH . BA.

PROBLEM 3.

The Hypotenusa and one leg given , to find the angles and the other leg.

IN any right angled Triangle let one of the legs be 512, and the hypotenusa 640. Draw a line at pleasure as A B, and by your Scale of equal parts, number from B to A the quantity of the leg given , then upon the point A , erect the perpendicular A C, and open your compasses to the extent of your hypotenusa given , and setting one foot in B, move the other till it touch the perpendicular A C , and there draw B C , so shall A C be the leg inquired , and either angle may be known by the line of chords , as hath been shewed already. *Fig.29.*

This Problem doth contein two other of the seven Cases in a right angled plain Triangle.

The hypotenusa and the leg given, to find { 4 The oblique angles. 5 The other leg.

The manner of protraction hath been fully enough declared in the first Problem , here therefore it shall suffice to set down the proportions in each Case , and first of the fourth Case.

4 Case.

The hypotenusa and a leg given, to find the oblique angles.

Therefore	by Sines	BC . BA :: BE . BD.
	by Secants	BA . BC :: BF . BG.
	by tang. & secants	BA . KH :: BC . KC.

5 Case

5 Case.

The Hypotenusa and leg given, to find the other leg.

This Case requireth two operations, the first is to find an angle by the Case preceding: the second is to find the leg inquired, either by the third Case laid down in the second Problem, or by the second Case, laid down in the first Problem.

PROBLEM 4.

The legs given to find the hypotenusa, and either of the oblique angles.

Fig. 29. IN any Right angled plain Triangle, let either of the given legs be 512. and the other 384. Draw a line at Pleasure as A B, and upon the point A, erect the perpendicular A C, and by help of your Scale of equal parts, set off from A to B 512, and also from A to C 384. and draw the line B C for the hypotenusa, which must be measured by the Scale of equal parts, and the oblique angles, as hath been shewed already.

This Problem doth contein the two last of the seven Cases in right angled plain Triangles, resolveable by these Axioms.

The legs given, to find	6	The oblique angles
	7	The Hypotenusa.

In the first of these Cases, the hypotenusa is neither given nor inquired, therefore the proportion must be by tangents onely: the manner of protraction hath been already shewed in the first Problem: the proportions are,

BA . CA : : BF . GF. Or thus, CA . BA : : CH . KH.

7 Case.

The legs given, to find the hypotenusa.

This Case requireth two operations, the first is to find an angle by the Case preceding; the second is to find

find the hypotenusa by the second Case, whose several proportions are laid down in the first Problem of this chapter, and need not be again repeated.

Hitherto we have spoken of Right angled plain Triangles, the Problems following concern such as are oblique angled, the resolution whereof by the line of chords & equal parts is plain and easie, but by the lines of sines and tangents more troublesome, we will however illustrate both, and first set down the two remaining Axioms, upon which the proportions for there resolution by numbers doe depend.

AXIOM 3.

In all plain Triangles, As the half sum of the sides, is to their half difference: So is the tangent of the half sum of their opposite angles, to the tangent of their half difference.

AXIOM 4.

In all plain Triangles; As the base is to the sum of the other side, So is the difference of those sides, to the difference of the segments of the base.

These two Axioms are for the solution of those Cases in oblique angled plain Triangles, in which the two former Axioms are deficient: for some of them may be resolved by the second Axiom onely, and one other case may be also resolved by the second, but not without help of the fourth: the Cases resolveable by the second Axiom onely are expressed in the two next Problems, and that Case in which the help of the fourth Axiom is required, we have reserved to the last place.

PROBLEM 5.

Two angles in an oblique angled plain Triangle being given, with any one of the three sides, to find the other sides, and the third angle.

Fig. 32. IN any oblique angled plain Triangle, let one of the given angles be 22 d. and the other 33.98. d. and let the given side be 670, the sum of the two angles given being deducted from a semicircle, leaveth the third angle: then draw the line CD at pleasure, and set off the given side 670 from C to D, and upon those points protract the angles adjacent to the given side, and draw the lines CE and DE, which must be extended till they intersect one another, to make the triangle CED, in which the sides unknown may be easily measured, as hath been shewed already.

In this problem there is conteined but one of the Cases in an oblique angled plain triangle, and that may be resolved by the second Axiom of this chapter. As in the oblique plain triangle CED let

The given	Side be	CD	670
	Angles	DCE	22 deg.
		CED	33.98 deg.

Then the proportion is, s CED . CD :: s ECD . DE

Fig. 33. To resolve this case by the line of Sines and equal parts, consider whether the side given be longer than the sine of the angle opposite thereunto or not.

If the side be longest, make AF equal to the given side CD 670 and opening your compasses in the line of Sines to the extent of the angle at E 33.98 deg. set one foot in F, and with the other describe the arch G, and draw the line AG, then take the sine of the angle C 22 deg. out of the same Scale, and setting one foot of that extent upon the line AF, removing it till it fall in such a place, as that the other foot being turned

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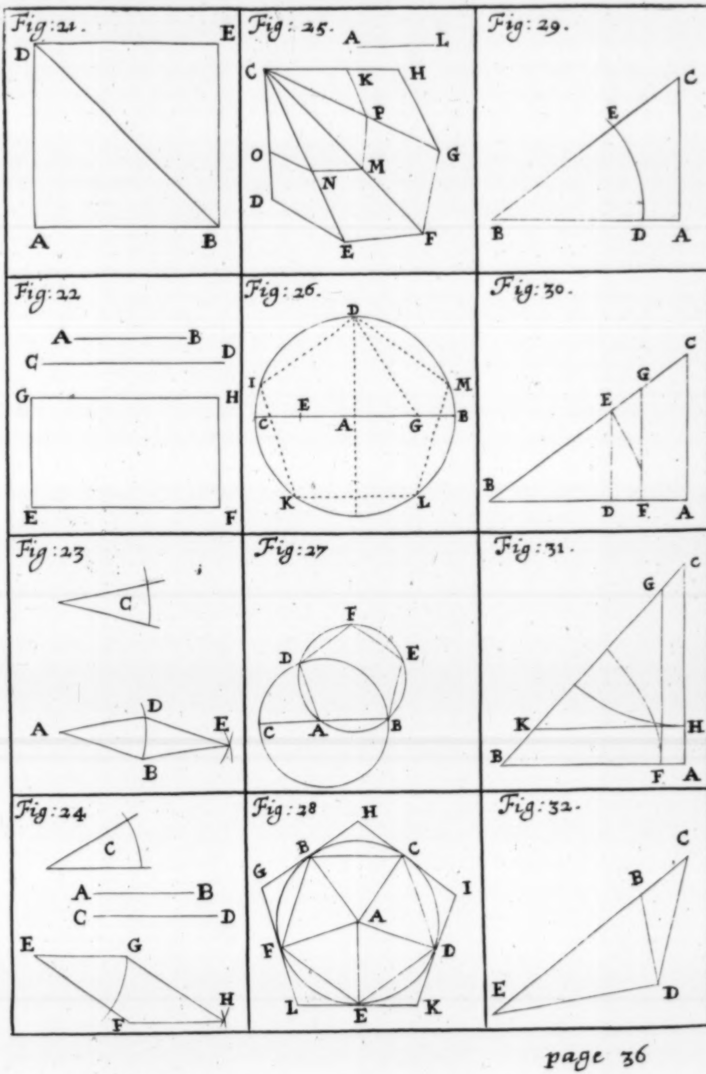

Fig:21.
Fig:25.
Fig:29.
Fig:22
Fig:26.
Fig:30.
Fig:23
Fig:27
Fig:31.
Fig:24
Fig:28
Fig:32.

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turned about will juſtly touch the line A G before drawn, and where upon ſuch conditions it reſteth, make the point H. Then meaſuring A H upon your Scale, you ſhall find it to reach 449 for the extent of D E the ſide inquired.

But if your Scale of Sines be of ſuch a length, as that the ſine of your angle doth exceed the length of your ſide oppoſite thereunto, make A F equal to the ſine of your greateſt angle given C E D 33.98 degrees, and A H equal to the angle C 22 deg. then opening your compaſſes to the extent of *C* D 670 in the line of equal parts, ſet one foot in F, and with the other deſcribe the arch G, and draw the line A G, laſtly, from H take the neareſt diſtance to the line A G, and that diſtance being meaſured upon the line of equal parts ſhall give you 449 as before.

PROBLEM 6.

Two ſides in an oblique angled plain Triangle being given, with an angle oppoſite to one of them, to find the other angles, and the third ſide, if it be known whether the angle oppoſite to the other ſide given be obtuſe or acute.

IN any oblique angled plain Triangle, let the given angle be 22 deg. and let the ſide adjacent to that angle be 670, and the ſide oppoſite 449, to find the other angles, and the third ſide: Upon the line D C from D to C, ſet off the length of the ſide adjacent to the given angle 670, and upon the point C, protract the angle given, & draw the line B C, then open your compaſſes to the length of the other ſide given 449, & ſetting one foot in D, turn the other about till it touch the line B C, which will be in two places in the point B, or the point neareſt to C if the angle oppoſite to the ſide *C* D be obtuſe, but in the point E, or the point furtheſt from *C* if acute: according therefore to the *Fig.* 32.

 ſpecies

species of that angle, you must draw either the line B D, or E D to compleat the triangle, and then you may measure the other angles, and the third side as hath been shewed.

Tis is another of the cases in an oblique angled plain Triangle which is resolveable by the second Axiom, and the Proportion is the converse of the former; *viz.* D E . s E C D :: C D . s C E D, if the angle sought be accute; but, D B . s E C D :: C D . s C B D if it be obtuse.

The solution of this Problem by the lines of sines and equal parts, as the same with the last Problem, and therefore needeth no further explanation.

PROBLEM 7.

Two sides of an oblique angled Triangle being given, with the angle comprehended by them, to find the rest.

Fig. 34. IN an oblique angled plain Triangle, let the given angle be 124 deg. let one of the given sides be 670, and 449 the other, to find the third side and the other angles, draw a line at pleasure as *B* D, and upon the point D, protract the given angle, and draw the line D *C*. Then open your compasses to the extent of 670 in the Scale of equal parts, and set them from D to *B*, and likewise to the extent of 449 in the same Scale, and set them from D to *C*. and draw the line *B C*, and so have you constituted the Triangle *B D C*, in which you may measure the angles and the third side, as hath been shewed already.

This Problem conteineth two other cases in oblique angled plain Triangles, *viz.*

Two sides and their contein-ed angle given, to find	3	The oblique angles.
	4	The third side.

The

The first of these cases must be resolved by the third Axiom: mechanically thus, Make A F equal to both the sides given, *viz.* B D and D C, and A H equal to the difference of the sides, then open your compasses in the line of Tangents from the beginning of the Scale to the tangent of 27.99 degrees, the half sum of the angles inquired, and setting one foot in the point F, with the other describe the arch G, and draw the line A G, then shall the nearest distance from H to the line A G taken in your compasses and measured upon the line of Tangents, be the tangent of 5.99 deg. half the difference of the angles inquired. *Fig.* 33.

Now then the half sum of B and C is	27.99 deg.
To which the half difference being added	5.99
Their sum is the angle B C D	33.98
Their difference is the angle C B D	22.00

And now having all the angles and two sides given, the third side may be easily found, as hath been already shewed in the fifth and sixth Problems.

PROBLEM 8.

The three sides of an Oblique angled Triangle being given, to find the angles.

LEt the length of one of the given sides be 407. The length of another 670; and the length of the third side 449. Draw a line at pleasure as B *C*, and by help of your Scale of equal parts, set off from *B* to *C* 670 the greatest side, then open your compasses in the same Scale, to the extent of either of the other sides, and setting one foot of your compasses in *B*, with the other describe an occult arch: lastly, extend your compasses in the same Scale to the length of the third side, as suppose 407, and at that extent of the compasses, *Fig.* 34.

upon

upon the point C, describe another arch, cutting the former in the point D, then will the lines *B* D and *C* D constitute and oblique angled Triangle, whose angles may be measured, as hath been already shewed.

To resolve this Problem by proportions, the fourth Axiom must be consulted; by help whereof making the longest side the base, the oblique angled Triangle given may be turned into two right angled Triangles in this manner.

Take the sum and difference of the two lesser sides, *B* D and D *C*, there sum is 856: there difference 42.

	Now then as the base B C	670
Fig. 34.	*Is to the sum of the sides* B D *and* D C	856
	So is the difference of the sides B D *and* D C	42
	To a fourth number, viz.	53.66
	Which number subtracted from the base B C 670, *the remainer is* C E.	616.34

Now then a line drawn from D to the middle of *C* E shall cut the line *BC* at right angles in the point F, and constitute the two right angled triangles *B*DF and DF*C*, in which we have given the hypotenusaes *B*D, and D*C*, and the legs *B*F and *C*F, and therefore we may find the angles of those Triangles, as hath been shewed in the third problem.

CHAP. VIII.

To find the superficial content of right lined Figures.

THe resolution or mensuration of Triangular plaines in respect of their sides and angles, hath been fully enough shewed in the former Chapters; we will now shew how the Area or Superficial

ficial content of them and any other plain figures may be found. And because all many sided figures may be best measured, by reducing them first into Right angled Triangles, Quadrangles, or Trapezias, we will first shew how the Area or Superficial content of these plain figures may be readily found.

PROPOSITION I.

To find the superficial content of a right angled parallelogram.

IF you multiply the two sides of a right angled parallelogram which comprehend a right angle by one another, the product shall be the content required.

But in a right angled Triangle, if you multiply the legs which comprehend the right angle by one another, half the product shall be the content required.

EXAMPLE.

Let the length of the parallelogram represented by A B *be*	45.64	*Fig.* 35.
And the breadth represented by A D *be*	23.16	
The content of the parallelogram is	1057.0224	
The half whereof	528.5112	
Is the superficial content of the triangle A D B.		

Another way to find the area or superficial content of a right angled triangle. Multlply the length A B by half the heigth A D, the product shall be the content of the triangle A B D, as before.

The length A B	45.64
Half the heigth AD	11.58
The content of ABD	528.5112

PROPOSITION 2.

Fig. 35. *To find the superficial content of a right angled Trapezium.*

Multiply half the sum of the parallel sides, by the nearest distance between them, the product shall be the content required. In the Right angled Trapezium A B C D, the side B C is 3.50, and the side A D parallel thereunto is 2.25, their sum 5.75, the half sum 2.875, the nearest distance or perpendicular A B 6.25, by which if you multiply 2.875, the product 17.96875 is the content of the Trapezium required.

Or thus:

If you multiply the sum of A D and B C 5.75 by half A B 3.125, the product will be 17.96875 the content, as before.

PROPOSITION 3.

To find the Area or superficial content of an oblique angled triangle.

Multiply half the heigth of the triangle by the base, or half the base by the heigth, and either way shall give you the content of the triangle.

EXAMPLE.

Fig. 34. In the oblique angled triangle B D C, the base B C is 20, and the height of the triangle is 6.6, the half whereof 3.3 being multiplied by 20, the product 66 is the content of the triangle, or the half of B C 10 being multiplied by F D 6.6 the product is 66, as before.

By another and more general way.

The three sides of a plain triangle being given, the area or superficial content may be thus found.

Adde the three sides together, and from the half sum subtract each side, and note their difference; then mul.

multiply the halfe ſum by the ſaid differences continually, the ſquare root of the laſt product, ſhall be the content required.

Example. In the triangle B C D, The ſide B C is 20, the ſide D B 13, and the ſide D C 11. The ſum of theſe three ſides 44, the half ſum 22, from whence ſubtract B C 20, the difference is 2. Again, from the half ſum 22 ſubtract D B 13, the difference is 9. And laſtly, ſubtract DC 11, the difference is 11. Again, the half ſum 22 being multiplied by the firſt difference 2, the product is 44, and 44 multiplied by the ſecond difference 9, the product is 396, and 396 multiplied by 11, the product is 4356, whoſe ſquare root 66, is the content required. *Fig.* 34.

PROPOSITION 4.

To find the Area or ſuperficial content of any oblique angular Quadrangle.

THe ſuperficial content of any oblique angled Quadrangle, may be found, by reducing the Quadrangle given into two triangles by a Diagonal drawn from one of the angles to another oppoſite angle: for if by help of perpendiculars let fall upon the this line from the other angles, you find the content of each triangle, as hath been ſhewed already, the ſum of the content of both triangles ſhall be the content of the Quadrangle.

Example. Let the content of the Quadrangle A B C D be required, in which let the diagonal D B be 121.64, the perpendicular F C 52.2, which being multiplied by the half of B D 60.82, the product will be the area of the triangle B C D 3174.804, In like manner, let the perpendicular E A be 21.6, which being multiplied by the half of B D 60.82, the product will be the content of the triangle A B D 1313.712. *Fig.* 36.

Now then if to the content of the triangle B C D	3174.804
You adde the content of the triangle A B D	1313.712
The content of the Quadrangle A B C D *will be*	4488.516

POPOSITION 5.

A plain irregular Multangle be given, to find the content thereof.

ALL plain irregular multangles which doe consist of many unequal sides, are commonly measured by reducing the multangle propounded into triangles, and finding the area of every triangle.

EXAMPLE.

Fig. 37. In the irregular multangled plain A B C D E F G H, if you draw the lines A D, D B, A E, A F, and F H, the whole figure will be reduced into six triangles; which being severally measured according to the former directions, and the content of all added together into one sum, will shew the content or area of the whole plain.

As suppose there were in the triangle B C D 72, A D B 84, A D E 110, A E F 121, A F H 165, and in F G H 66: these six numbers being added together doe make 618, which is the content or area, as was required.

PROPOSITION 6.

To find the content of any regular plain.

MUltiply half the circumference of the figure by the right line drawn from the center of the figure to the middle point of one of the sides, the product shall be the content required.

And to find the center of any regular figure, doe thus; Consider whether the figure given have an equal or unequal number of sides, if the number of sides in the figure be equal, draw a line from one angle to another

another angle opposite to it, and divide that line into two equal parts, the middle point of section shall be the center of the figure, but if the number of the sides be odd, you must draw two right lines, from the middle of two sides to their opposite angles, where these lines shall intersect one another, shall be the center of the figure. As in the Pentagon the lines E B and D C doe mutually intersect one another in the point A, which is the center of the figure: now then let the line A D or A E be 14.63, and one side of the figure 17.2, half the circumference will be 43, which being multiplied by the right line D A 14.63, the product 629.09 is the area or content thereof. *Fig. 38.*

The length of the line which is drawn from the center of any figure to the middle of one of the sides, may *Arithmetically* be thus found, by the number of sides in the figure propounded, divide 360 the number of degrees in a circle, the quotient shall be the angle at the center C A B, the half whereof is the angle F A E, which being known, together with F E the half of F G, the line A E is easily found.

For, t F A E. F E : : Rad. A E, *by the 1 Probl. of right angled triangles.*

PROPOSITION 7.

The radius of a circle being given, to find the side of any figure inscribed in the same circle, or the contrary: the side of any figure inscribed in a circle being given, to find the radius of the circle circumscribing the figure given.

LEt the Radius of the circle in which the pentagon B C F G D is inscribed, be A C 746, and let the side of that figure be inquired.

 The

The angle F A E may be found as in the last *Probl.* Then say, Rad. A F, Sine F A E . F E, whose double is F G inquired.

In like manner one of the sides being given, I say, Sine F *A* E . F E $\frac{1}{2}$ F G : : Rad. *A* F.

CHAP. IX.

Of curved or circular lines, and the superficies of a Circle.

HItherto we have spoken of right lines and right lined plains, and now we are to speake of curved or circular lines, in which we might proceed as we have already done in the former; but that is not our present purpose, we shall reserve that for another Treatise hereunto annexed, (*viz.* for our Description and use of the Globes, to which it properly belongeth) and shall not treat any further of circular lines here, then will be expedient or necessary for the finding of the Circumference, Diameter, and Area or Superficial content of any circle given, with the solidity of any regular body, whether it be composed of right lined, or curved lined superficies.

1 And to that purpose it will be necessary that the Diameter, Circumference, and Area of some one circle be given, that from any one of these being given in another, the rest may be found.

2 The Diameter of a circle in general is now generally supposed to be 2 with cyphers, and according to this supposition, our Canons of Sines, Tangents, and Secants; natural and artificial are composed.

3 The Diameter of a circle being 2 with cyphers, the circumference of that circle is found by multiplying the sine or chord of the least arch for which the Canon

Canon of Sines is made, by the number of parts into which the whole circle is ſuppoſed to be divided. Thus in the Canon of Sines which Mr. *Briggs* compoſed, the circle is ſuppoſed to be divided into 360 parts or degrees, and every degree into 100 parts: that is the circle is ſuppoſed to be divided into 36000 parts, and the ſine of one of theſe parts, that is, the ſine of one hundredth part of a degree by that Table is 174532924313, Which being multiplied by 36000, The product 6.283185275288000, is the circumference of a circle whoſe Diameter is two with cyphers.

4 The Diameter and circumference of one circle being thus aſcertained, we ſhall as *Archimedes* hath ſufficiently proved, take for granted, that every circle is equal to a right angled plain triangle, whoſe legs comprehending the right angle, are one of them equal to the Semidiameter, and the other to the circumference of that circle. And therefore the area or Superficial content of a circle may be found by multiplying half the circumference by half the Diameter, or the whole Diameter by the fourth part of the circumference, thus taking the Diameter of a circle to be 14, and the circumference 44, whether you multiply the half circumference 22 by 7, half the Diameter, or the whole Diameter 14, by 11, the fourth part of the circumference, the product and Area will be 154. In like manner, if you take the Diameter of a circle to be 1.00000, and the circumference to be 3.14150, the product of half the circumference, 1.570795 being multiplied by half the Diameter, .50000 will be .7853975, or the fourth of the circumference .7853975 being multiplied by the whole Diameter 1.00000 will be the ſame ſtill.

And hence the ſuperficies of any other circle may be found by the proportions in the following Problems, or Propoſitions.

PROPOSITION 1.

The Diameter of a circle being given, to find the circumference.

AS 1 is to 3.14159, So is the Diameter, to the circumference: and therefore if you multiply this number 3.14159 by the diameter given, the product is the circumference you look for.

EXAMPLE.

Fig. 39. The diameter A D 28.25 being multiplied by 3.14159 the product 88.7499175 is the circumference of that circle.

But this and the following propositions may be most easily performed, by artificial numbers or Logarithms, whose construction and use we have shewed in our *Trigonometria Britannica*, for a more full understanding whereof we must refer the Reader thither; here onely he may Note, that multiplication is performed by Addition, and Division, by Subtraction, as may be seen in all the Examples following.

As 1	0.00000000
Is to 3.14159	0.49714986
So is the Diameter A D 28.25	1.45101845
To the circumference 88.749	1.94816831

PROPOSITION 2.

The Diameter of a circle being given, to find the superficial content.

AS 1 to .785 3975, So is the square of the diameter, to the superficial content, and therefore, if you multiply 785397 5 by the square of the diameter, the product will be the content required.

In Artificial numbers, the Logarithm of any number doubled is the Logarithm of the square of that number, therefore the Logarithm of the diameter being

being twice added to the Logarithm of .7853975 the sum or aggregate shall be the Logarithm of the superficial content required.

EXAMPLE.

As 1, Is to .7853978	9.89508988
So is the Diameter A D 28.25	1.45101845
	1.45101845
To the content required 626.79	2.79712638

PROPOSITION 4.

The Diameter of a circle being given, to find the side of a square which may be inscribed within the same circle.

THE Chord or subtense of the fourth part of a circle (that is of 90) whose diameter is 1, is .7071067, now then the proportion is: As 1, is to .7071067, So is the diameter of another circle to the side required; and therefore, If .7071067 be multiplied by the diameter given, the product will be the side of the square you look for.

Thus in the circle of the first proposition, the diameter A D being 28.25 B E the side of the square, B C F E will be found to be 19.975.

As 1, Is to .7071067	9.84948500
So is the Diameter A D 28.25	1.45101845
The side required 19.975	1.30050345

PROPOSITION 4.

The circumference of a circle being given, to find the Diameter.

BY the diameter to find the circumference, the proportion (by the first prop.) was, as 1, to 3.14159, so is the diameter to the circumference: therefore also

as 3.14159, is to 1, so is the circumference to the diameter, if therefore you make the circumference to be 1. I may say, as 3,14159, is to 1, so is 1, to .318308, and hence to bring an Unite in the first place. I say, as 1, to .318308, so is the circumference to the diameter.

And therefore, if you multiply .318308 by the circumference, the product will be the diameter required.

EXAMPLE.

As 1, is to .318308	9.50285014
So is the circumference given 88.749	1.94816831
The diameter required 28.25	1.45101845

PROPOSITION 5.

The circumference of a circle being given, to find the Superficial content.

As the square of the circumference of a circle given, is to the superficial content of that circle also given; so is the square of the circumference of another circle given, to the superficial content required.

Example. In a circle whose diameter is 1, the circumference is 3.14159, and the area .7853978, and let the circumference of the circle A B C D F E be 88.75, I say,

As the square of 3.14159 the Logar.	0.49714986
The logarithm doubled	0.99429972
To the superficial content 7853978	9.89508988
So is the square of .88.749	1.94816831
	1.94816831
	3.79142650
The content required 626.79	2.79712678

But for the more ease in working, an Unity may be brought into the first place, by finding a number proportional to the numbers given, thus,

As

As the logar. of the square of 3.14159	0.99429972
Is to the logarithm of .7853978	9.8950888 8
So is a square of 1	0.0000000 0
To the fourth proportional .079578	8.90079016

Now then we may say, As 1, is to .079578; so is the square of the circumference to the superficial content. And therefore, If .079578 by multiplied by the square of the circumference, the product will give you the content you look for, as before.

EXAMPLE.

As 1, *is to* .079578	8.9007 016
So is the square of 88.749	1.9 16831
	1 94 16831
To the superficial content 6 6.79	2.79712678

PROPOSITION 6.

The circumference of a circle being given, to find the side of the square which may be inscribed within the same circle.

AS the circumference of a circle whose Diameter is 1, *viz.* 3.14159, is to the side of the square which may be inscribed in the same circles, *viz.* .707107: so is the circumference of another circle, to the side inquired; and to bring an Unity in the first place, we will suppose the circumference of a circle to be 1, and according to this rule, the side inquired may be thus found.

As the logarithm of 3.14159	0.49714986
Is to the logarithm of .707107	9.849 8500
So is the logarithm of 1	0.00000000
To the side inquired .225078	9.35233514

Now then, as 1, is to .225078, so is the circumference to the side inquired: and therefore, if .225078 be multiplied by the circumference, the product will be the side of the square demanded.

EXAMPLE.

As 1 is to	.225078	9.35233514
So is the circumference given	88.749	1.94816831
To the side required BE	19.975	1.30050345

PROPOSITION 7.

The superficial content of a circle being given, to find the Diameter.

THis is the Converse of the second proposition, The Diameter given to find the content, for which the proportion is. As 1, is to .7853978: so is the square of the Diameter, to the superficial content; and therefore to find the Diameter we must say: As .7853978, is to 1, so is the content, to the square of the Diameter: and to bring an Unity into the first place, say,

As the logarithm of	.7853978	9.89508988
Is to	1	0.00000000
So is	1	0.00000000
To the fourth number	1.27324	0.10491012

Now then, as 1, is to 1.27324, so is the content to the square of the Diameter: and therefore if you multiply 1.27324 by the content given, the square root of the product will be the Diameter required.

EXAMPLE.

As 1, is to	1.27324	0.10491012
So is the content given	626.79	2.79712678
		2.90203690
To the Diameter required	28.25	1.45101845

PROPOSITION 8.

The ſuperficial content of a circle being given, to find the circumference.

THis is the converſe of the fifth propoſition, The circumference given, to find the content; for which the proportion is, as 1, is to .079578, ſo is the ſquare of the circumference, to the ſuperficial content: and therefore, to find the circumference, we muſt ſay: as .079578, is to 1, ſo is the ſuperficial content, to the ſquare of the circumference; and to bring an Unity in the firſt place, as .075578, is to 1, ſo is 1, to 12.5664: and therefore, if 12.5664 be multiplied by the content, the ſquare root of the product will be the circumference required.

EXAMPLE.

As 1, is to 12.5664	1.09920984
So is the content given 626.79	2.79712678
	3.89633662
To the circumference required 88.749	1.94816831

PROPOSITION 9.

The ſuperficial content of a circle being given, to find the ſide of a ſquare equal unto it.

EXtract the ſquare root of the content given, this done, that root is the ſide required.

EXAMPLE.

Let the content given be 626.79	2.79712678
The ſide required 25.035	1.39856339

And in this manner, the ſuperficies of any other figure whatſoever, or of more figures then one, may be reduced into a ſquare, if (according to this Example) you firſt find the ſide of the content, as of a given ſquare.

PRO-

PROPOSITION 10.

The Axis or Diameter of a Sphere being given, to find the ſuperficial content

AS the ſquare of the Diameter of a circle, which ſuppoſe 1, is to the ſuperficial content thereof 3.14159, ſo is the ſquare of the Axis or Diameter given, to the ſuperficial content required.

EXAMPLE.

The logarithm of 3.14159	0.49714986
Logar. of the given Axis 28.25	1.45101845
	1.45101845
The ſuperficial content 2507.1	3.39917676

CHAP. X.

Of Bodies, or Solids.

AFter the deſcription of lines and planes, the doctrine of Bodies is to be conſidered.

1 A *Solid* or *Body*, is that which hath length, breadth, and thickneſs, whoſe bounds or limits are ſuperficies.

2 A *Solid* is either plain or Gibbus.

3 A plain *Solid* is that which is comprehended of plain Superficies, and is either a *Pyramide*, or a *Pyramidate*. A *Pyramide* is the firſt kind of Solid, becauſe the moſt ſimple, as a Triangle, is the moſt ſimple plain: and as all plaines may be reduced into Triangles ſo all bodies may be reduced into Pyramides, and therefore are called Pyramidates, as though they were made of Pyramides.

4 A *Pyramide* is a Solid figure, which is conteined by ſeveral plains ſet upon one plain or baſe, and meeting

ing in one point. This is a general Definition of a Pyramide, whether the base be a Triangle, Quadrangle, or any other right lined plain. A particular Pyramis therefore taketh its name from the figure of the base thereof. Thus that is called a Triangular Pyramis, whose base is a Triangle, that a Quadrangular whose base is a Quadrangle, and so of the rest.

5 Of all the several sorts of Pyramides, there is but one that is regular, to wit, a *Tetrahedron*, or a Pyramide consisting of four regular or equilateral Triangles: the form whereof. (as it may be cut in pastboard) may be conceived by figure 40.

6 A *Pyramidate* is a Solid figure composed of Pyramides, and is either a *Prisme*, or a mixt *Polyhedron*.

7 A *Prisme* is a Pyramidate, or solid figure, conteined by plains; of which those two which are opposite are equal, like and parallel, and all the other are parallelograms.

A *Prisme* is either a *Pentahedron*, a *Hexahedron*, or a *Polyhedron*;

A *Pentahedron Prisme*, is that which is comprehended of five sides, and the base a Triangle.

An *Hexahedron*, it that which is comprehended of six sides, and the base a Quadrangle.

A *Polyhedron*, is that which is comprehended of more then five sides, and the base a Multangle.

9 If Right angled Parallelograms of equal heigth shall be drawn upon every side of a Triangle, and to the extremity of one of the parallelograms there shall be made a Triangle equal & like to the former, of these set in their due places thou mayest make a *Pontahedron Prisme*, as in the 41 figure.

10 A *Polihedron Prisme* may be made, by drawing right angled parallelograms on every side of a multangled plain; and in the extremity of one of the parallelograms another multangled plain equal and like to the other.

11 An *Hexahedron Prisme* is distinguished into a

Paral-

Parallelipipedon and a *Trapezium*. A *Trapezium* is that whose opposite plains or sides are neither parallel nor equal.

A *Parallelipipedon*, is that whose sides or opposite plains are parallelograms, and therefore parallel and equal.

12 A *Parallelipipedon*, is either right angled, or oblique.

13 A Right angled *Parallelipipedon* it that, which is comprehended of right angled sides; and it is either a Cube or an Oblong.

14 A *Cube* is a right angled Parallelipipedon of equal sides.

15 An *Oblong* is a right angled Parallelipipedon of unequal sides.

16 An Oblique angled parallipipedon, is that which is comprehended of oblique sides, and is either a *Rhombus*, or a *Rhomboides*.

And hence it is manifest, that there are as many sorts of Parallelipipedons as of parallelograms: for if there be made six right angled equilateral parallelograms, that is, six square solid angles, there shall be made a Cube, answerable to a square in plains: and amongst the Prismes, this onely is regular, as the square is amongst the Quadrangles. Moreover, if six Right angled parallelograms, and not of equal sides, but longer in one part, though two of them being equilateral, be composed together, it shall be called a long parellelipipedon. But if six oblique angled equilateral parallelograms shall be joyned together, they shall be called a *Rhombus*: lastly, if six oblique angled and inequilateral parallelograms be composed, though the two of them, (which are opposite to one another) be equal, such a parallelipipedon shall be called a *Rhomboides*.

17 A mixt *Polyhedron*, is that whose Vertex is in the center, and the several sides exposed to view and of this sort there are onely three, viz. the *Octohedron*, the *Icosahedron*, of both which the base is a Triangle, and the *Dodecahedron*, whose base is a Quincangle.

Fig: 33.

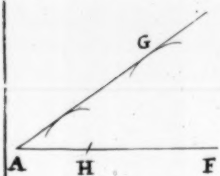


Fig: 37.

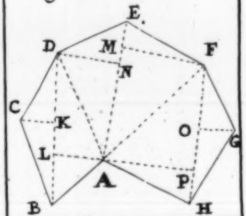


Fig: 41.

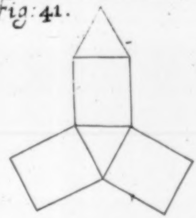


Fig: 34.

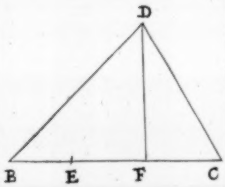


Fig: 38.

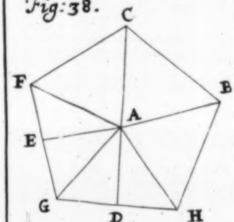


Fig: 42

Cube

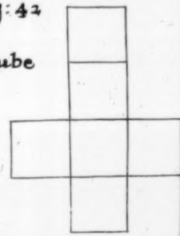


Fig: 35.

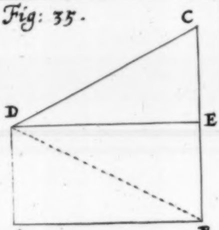


Fig: 39.

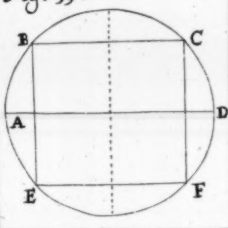


Fig: 43

Octohedron

Fig: 36.

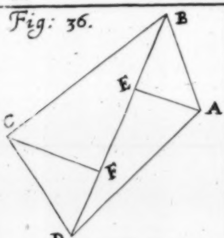
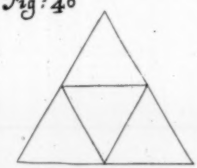


Fig: 40



Tetrahedron

Fig: 44

Icosahedron

page 56

18 An *Octohedron* is a ſolide figure, which is contein-ed by eight equal and equilateral Triangles

19 An *Iſoſahedron* is a ſolid figure, which is conteined by twenty equal and equilateral Triangles.

20 A *Dodecahedron* is a ſolid figure, which is contein-ed by twelve equal pentagons, equilateral and equi-angled.

An *Octohedron* may be made, by cutting eight equal triangles. As in figure 43.

An *Iſoſahedron* is made with 20 equal regular Tri-angles rightly diſpoſed. As in fiigure 44.

A *Decocahedron* is made of twelve regular pentagons. *Fig.* 45.

21 A *Gibbus Solid*, is that which is comprehended of *Gibbus ſuperficies*, and it is either a *Sphere* or *Various*.

22 A *Sphere* is a Gibbus body abſolutely round and Globular. See in figure 46.

23 A *Various* Gibbus body, is that which is compre-hended by Various ſuperficies and a circular baſe: and it is either a *Cone* or a *Cylinder*.

24 A *Cone* is a Piramidical body, whoſe baſe is a Circle. See figure 47.

25 A *Cylinder* is a ſolid body of equal thickneſſes, whoſe baſe is a Circle. See figure 49.

PROPOSITION 1.

The baſe & altitude of a Pyramide or Cone given to ſind the ſolid content.

Multiply the altitude by a third part of the baſe, or whole baſe by a third part of the altitude, the product ſhal be the ſolid content thereof.

Example. In a Pyramide having a Quadrangular baſe, whoſe altitude AB 31.5 being given, the third part thereof is 10.5, and the baſe 564.08 found by mul-tiplying the Diagonal 128.2, by the perpendicular 4.4. *Fig.* 48.

Now then if you multiply 564.08
By the third part of the altitude 10.5

282040
56408

The product 5922840
Is the solid content of the Pyramide.

Fig. 47. *Example.* In a Cone whose Diameter being given A B 2.5 the base *by the 2 Prop. of the 9 chap.* will be 96.25, and let the altitude of the Cone C D be 16.92, the third part whereof is 5.64: now then if you multiply CD 96.25 by 5.64, the product 542.85 is the solid content thereof.

PROPOSITION 2.

The base of a Prisme or Cylinder being given, to find the solid content.

MUltiply the base of the Prisme or Cylinder given, by the altitude; the product shall be the solid content required.

Example. In an Hexahedron Prisme one side of whose base being 12.52, and the other 5.3 the superficies or base it self will be 66.356, which being multiplied by the given altitude 502 the product 333.10712 is the solid content required.

Example. In a Cylinder: whose base let be equal to the base of the Cone given 69.25, and let the altitude be 16.92: Now then if you multiply 96.25 by 16.92 the product 162.855 shall be the solid content thereof.

PROPOSITION 3.

The two bases of an irregular solid being given, to find the solid content thereof.

ALthough there be infinite varieties of irregular solids, yet here I call them onely irregular which being

being considered longwise are strait, and have one base greater then the other, such as are the solids in figure 50, in which you way observe, that the bases A B C D are longer then the bases F F G H: now to find the solid content of such solids, this is the Rule.

Multiply the greater base by the lesse, and of that product extract the square root, this done, if (by the aggregate of that root and both the bases) you multiply the third part of the length of the solid propounded, the last product is the solid content required.

Let the given length be 15.24 the third part is 5.08.

The greater base	1.92	0.28330123
The lesser base	.85	9.92941893
The product	1.6320	0.21272016
The square root	1.2775	0.10636008
The greater base	1.92	
The lesser base	0.85	
The root found	1.2775	
Their aggregate	4.0475	0.60718686
The third part of the length	5.08	0.70586371
The solid content required	20.561	1.31305057

PROPOSITION 4.

In a piece or frustrum of a pyramide or Cone both tht bases and the altitude being given, to find the content thereof.

IF the aggregate of both the bases of the frustrum, and of the mean proportional between them, be drawn into the altitude of the Frustrum: the third part of the product shall be equal to the solid content of the frustrum.

Example. Let A B C D E F represent the lesser base of the frustrum, H K L M the greater, N P the altitude given, let the greater base of this frustrum be the area of a regular Hexagon whose side is 12, *viz.* 374.4, and the lesser base the area of a regular Hexagon whose side is 6, *viz.* 93.6, to find the mean proportional between these two bases, I multiply 374.4 by 93.6, the product is 35043.84, whose square root 187.2 being added to the two bases given, the sum or aggregate is 655.2, which being multiplied by the altitude supposed to be given P N 15, the product 9828 is the concentent of the frustrum tripled, and therefore 3276 the the third part thereof, is the true content of the frustrum as was required.

PROPOSITION 7.

The diameters of a Vessel at the Head and Bung, with the length thereof being given, to find the content.

BY the Diameter given find the Area or superficial content of there circles as hath been shewed already: Then adde together two third parts of the superficial content of the greater circle, and one third part of the lesse: and there aggregate multiply by the length, the product shall be the content of the Vessel inquired.

Example. Let the Diameter of a Vessel at he Bung be 32, and at the head 18, and the length 40. The superficial content of a circle whose diameter is 32, will be 804.249 and the superficial content of a circle whose Diameter is 18 will be found to be 254.469, Two thirds of 804.249, the greater circle is 536.166, and one third of 254.469 the lesser is 84.823, the sum or aggregate of these two is 620.989, which being multiplied by the length given, *viz,* 40 the product 24839.560 is the content inquired.

PRO-

PROPOSITION 6.

The Axis of a Sphere being given to find the ſolid content.

A Sphere (as *Archimedes* hath proved) is equal to two thirds a Cylinder circumſcribing it; now then becauſe ſuch a Cylinder is made, if the area of a circle be multiplied by the Diameter: therefore of the area of a circle ſhall be multiplied by two thirds of the Diameter, the product ſhall be the ſolid content thereof.

The area of a circle whoſe Diameter is 1, was found before to be .785397, which being multiplied by .666666 the two thirds of the Diameter, the product .523598 is the ſolid content of the Sphere, hence to find the ſolid content of another Sphere whoſe axis is given, I ſay,

As the Cube I the axis given, is to .523598, ſo is the Cube of another axis given, to the ſolid content required.

EXAMPLE.

Let the given Axis be	28.25	1.45101845
The Cube thereof is	22545	4.35305535
The logarithm of .523598 *is*		9.71899862
The ſolid content required	11804.	4.07205397

CHAP. XI.

Of the meaſuring of the five regular Bodies.

THere are in Nature (as hath been ſaid) five plain regular Bodies: I The *Tetrahedron*. II The *Hexahedron* or *Cube*. III The *Octohedron*. IV The *Dodecahedron*. And V The *Icoſahedron*.

 The

The two first of these, *viz.* the *Tetrahedron*, and the *Hexahedron*, may be easily measured, as hath been already shewed, the one, *viz.* the *Tetrahedron* being a regular *Pyramide*, and the other a right angled *Parallelipipedon* of equal sides: for measuring of the other three, another method must be prescribed, which is indeed somewhat troublesome: but withal more general; in the *Octoedron*, *Dodecaedron*, and *Isosaedron*, the trouble cannot be avoided, and therefore for conformity sake, we will make use of one and the same method in all.

1 The five regular Bodies are composed of *Pyramides* that are equal, and of equal heigth, whose bases doe appeare without, but their vertical points doe concurre or meet within the center.

2 The altitudes or heigths of these *Pyramides* are equal to the perpendiculars drawn from the center of the body, to the center of the base, or to the Radius of the Sphere Inscribed within the same body. And it hath been already said, that if the heigth of a *Pyramide* shall be multiplied by a third part of the base, that the product shall give the solid content: and therefore the product found by the multiplication of the Radius of the inscribed Sphere, by the third part of the superficies of any Body, shall be equal to the solid content of the body; so that the trouble in measuring of these Bodies, doth chiefly consist in the finding of the Radius of the inscribed Sphere.

3 And if these five regular bodies shall be inscribed in the same Sphere, the same circle shall circumscribe the Triangle of the *Icosaedron*, & the Quincangle of the *Dodecaedron*, as also the Triangle of the *Octoedron*, and the Quadrangle of the *Cube*. And the solidities of these Bodies shall be proportional to their superficies.

4 The solid content of the *Tetrahedron*, hath such proportion to the solid content of the Cube, circumscribed by the same circle: as the side of the Equilateral Triangle in the *Tetrahedrons* base, hath to the Diameter of the circumscribing circle.

5 The

5 The ſolid content of the *Icoſaedron*, is to the *Dodecaedron*, as the ſide of the *Icoſaedron* is to the ſide of a *Cube* inſcribed in the ſame circle.

And hence the ſides of the five regular Bodies inſcribed in the ſame Sphere being given, we ſhall lay down a ſhort and clear method for the finding of the Radius, of their inſcribed circle, and conſequently of their ſolid content.

PROPOSITION I.

The ſide of any Tetrahedon being given, to find the Radius of his inſcribed Sphere, with the ſuperficial and ſolid content.

THe ſide of a *Tetrahedron* inſcribed in a Sphere' whoſe Radius is an Unity, Mr. *Briggs* in his *Arithmetica Ligarithmica chap.* 32. hath ſhewed to be 1.63299316̶18, the Radius of whoſe inſcribed Sphere, or any other ſide given, may thus be found.

Multiply the ſide given by it ſelf, and the product divide by 24 the ſquare root of the quotient, is the Radius of the inſcribed Sphere.

EXAMPLE.

The ſide of the Tetrahedron given is	1.6329931618
The logarithm of that ſide is	0.21298437
The log. doubled is the log. of the ſquare thereof	0.42596874
From which deduct the logarithm of 24	1.38021124
The remainer is the logar. of the Radius ſquare	9.04575750
The half whereof is the logar. of the Radius	9.52287855

Having thus found the Radius of the inſribed Sphere or altitude of each of the four Pyramides in the Tetrahedron, whoſe content is ſought, we muſt next ſeek the ſuperficial content thereof: thus,

The

The base of every pyramide is an equilateral Triangle, whose superficies may be found, *by the 6 Prop. of the 9 Chap.* and in our Example will be 1.1547005384, which being multiplied by 4, giveth the superficies of the whole body 4.6188021536

The third part thereof, is 1.5396007178

Which being multiplied by the Radius of the inscribed Sphere, giveth the solid content

Examp. The logarithm of the superficial content found, 4.6188021536, is 0.66452935, from which deducting 0.47712125, the logar. of 3 the remainer 0.18740810 is the logarithm of 1.539600717

To whose logatithm 0.18740810

Adde the logarithm Radius of the inscribed Sphere } 9.52287875

The aggregate is 9.71028685

The logarithm of 5.132002393 the solid content required.

And the solid content of any one *Tetrahedron* being thus found, the solid content of any other, may be readily found in this manner. As the Cube of the side of the *Tetrahedron*, whose solid content is given, is to the solid content thereof, so is the Cube of another Tetrahedron, to the solid content thereof.

Therefore, *if you multiply the solid content given, by the Cube of the side propounded, and divide the product by the Cube of the side given, the quotient shall be the solid content required.*

Example. From the side given 1.6239931618, the solid content was found to be 5,132002393. Hence from another given side, *viz.* 1, to find the solid content, I say,

As the Cube of the first side given	0.63895391
Is to the solid content thereof	7.71028658
So is the Cube of 1	0.00000000
To the solid content .11785	9.07133294

Therefore

Therefore as 1, is to 11785, so is the Cube of the side given, to the solid content thereof required.

Example. Let the side of a Tetrahedron given, be 12.

As 1	0.00000000
Is to the logarithm of .11785	9.07133295
So is the Cube of 12, viz. 1728	3.23754374
To the solid content 203.6467	2.30887641

PROPOSITION 2.

The side of an Hexahedron or Cube being given to find the Radius of his inscribed Sphere, with the superficial and solid content.

THe side of an Hexahedron inscribed in a Sphere (whose Radius is an unity) is 115470053 84, and the half of this side 7773502692 is the Radius of the Sphere inscribed in that body.

The square of this side is the superficial content of one base, and that content doubled, is the superficies of the whole figure, which being multiplied by the Radius of the inscribed Sphere, giveth the solid content required.

EXAMPLE.

The given side is .11547005 *logar.*	0.06246937
The square thereof is	0.12493874
To which adde the logarithm of 2	0.30102999
The square doubled	0.42596873
The logar. of the radius of the inscribed Sphere .5773502	9.76143938
The solid content required	0.18740811

POPOSITION 2.

The side of an Octohedron being given, to find the Radius of his inscribed Sphere, with the superficial and solid content.

THe side of an *Octoedron* inscribed in a Sphere, whose Radius is an Unity, is 1.4142135, and the Radius of the inscribed Sphere is the same with that in the *Hexahedron*; Hence therefore to find the solid content of this Body, you must first find the superficial content of one of the bases thereof, as hath been shewed in the *6 Prop. of the 9 Chapter*, which being a regular Triangle in our Example will be .866025404, and the superficial content being multiplied by 8, because this figure hath 8 Triangular bases,

The whole superficies will be	6.928203272
And the third part	2.309401067

The logar. of the $\frac{1}{3}$ of the superficies is	0.36349937
The logar. Radius of the inscribed Sphere	9.76143938
The solid content is 1.33333333	0.12493875

And the side of any other *Octohedron* being given, the solidy thereof may be readily found in this manner. As the Cube of 1.4142135, is to the solid content found 1.3333333: So is the Cube of any other side given, to the solid content inquired.

And hence to bring an Unity in the first place.

First Cube the given side 1.4142135	0.15051500
As the Cube of the side	0.45154500
Is to the solid content 1.3333333	0.12493873
So is the Cube of 1, *to the solid content*	9.67339373

And then the solid content of any *Octohedron* suppose of 1, whose side is 12, may thus be found.

As

As the Cube of 1	0.00000000
Is to the solid content	9.67339373
So is the Cube of 12, viz. 1728	3.23754374
To the solid content 814.5878	2.91093747

PROPOSITION 4.

The side of an Icosaedron being given, to find the Radius of his inscribed Sphere, with the superficial and solid content

THe side of an *Icosaedron* inscribed in a Sphere whose Radius is an Unite, is (as Mr. *Briggs* hath shewed) 1.051462247, and the Radius of the inscribed Sphere, is the Root universal of the Root of $\frac{4}{45}$ of the Radius of the Sphere given added to $\frac{1}{3}$ of the same Radius. Now $\frac{4}{45}$ of the radius of the Sphere given is .888888888 the Root whereof is .29814, to which $\frac{1}{3}$ of the Radius given being added, *viz.* 33333, the sum is 63147, and the Root of this sum .794654 is the Radius of the inscribed Sphere inquired.

Hence to find the solid content of this Body, the superficial content of one of the Triangular bases must be inquired, *by the 6 of the 9 hereof*, and in our Example will be .4787270692 which being multiplied by 20, because this body hath 20 Triangular bases, the superficies of the whole Body will be 9.574541384, and the third part thereof 3.191513774.

The logarithm of 3.1915137 *is*	0.50399582
The logar. of the rad. of the inscribed sphere	9.90017834
The solid content is 2.53615071	0.40417416

And the side of any other *Icosaedron* being given, the solidity thereof may be readily found in this manner.

As the Cube of the side given	0.06538102
Is to the solid content thereof	0.40417416
So is the Cube of 1, *to the content thereof*	0.33879314

And the ſolid content of any *Icoſaedron* ſuppoſe of 1, whoſe ſide is 12, may thus be found.

As the Cube of 1	0.00000000
Is to the ſolid content	0.33879314
So is the Cube of 12, viz. 1728	3.23754374
The ſolid content 3769.9	3.57633688

PROPOSITION 5.

The ſide of a Dodecaedron being given, to find the Radius of his inſcribed Sphere, with the ſuperficial and ſolid content.

THe ſide of a *Dodecaedron* inſcribed in a Sphere, whoſe Radius is an Unite, is (as Mr. *Briggs* hath ſhewed) .713644, and the Radius of the inſcribed Sphere, is the ſame with that in the *Icoſaedron*. Now then to find the ſolid content of this Body, the ſuperficial content of one of the Pentagonal baſes muſt be inquired *by the* 6th *of the* 9th *hereof*, and in our Example will be .876218520 which being multiplied by 12, becauſe this Body hath 12 Pentagonal baſes, the ſuperficies of the whole Body will be 10.514622242, and the third part thereof is 3.50487408.

The logarithm of 3.50487408 *is*	0.54177242
The logar. Rad. of the inſcribed Sphere	9.90017834
The ſolid content 2.78516	0.44485076

And the ſide of any other *Dodecaedron* being given, the ſolidity thereof may be readily found, in this manner:

As the Cube of the ſide given	9.56044519
Is to the ſolid content thereof	0.44485076
So is the Cube of 1, *to the content*	0.88440557

And

And then the ſolid content of any *Dodecaedron* ſuppoſe of 1, whoſe ſide is 6, may thus be found.

As the Cube of 1	0.00000000
Is to the ſolid content thereof	0.88440557
So is the Cube of 6, viz. 216	2.33445374
The ſolid content 16542	4.21885932

CHAP. XII.

The Deſcription and uſe of the Carpenters Rule.

THe Carpenters Rule is in length generally two foot, or 24 inches, according to the Standard, each inch being divided into 8 parts, that is, into halves, quarters, and half-quarters: the half inches are known from the quarters, and quarters from the half-quarters, by ſhort, longer, and longeſt ſtrokes, and at every whole inch is ſet figures, proceeding from 1 to 24, from the right hand towards the left, thoſe parts and figures, are ſet upon both edges, of one ſide the Rule, and are numbred both wayes, that however you hold the Rule on that ſide it may be ſtill ready for your uſe.

On the other ſide you have the lines of Board and Timber meaſure, the conſtruction whereof ſhall be ſhewed in the following Problemes, the uſe is thus.

Firſt, *Of the line of Board meaſure.*

IF the breadth of any ſuperficies (as Board, Glaſſe, or the like) be given in inches and part of an inch, you may by this line find how much of that inlength will make a foot ſquare, that is, how much of that ſuperficies

cies inlength, is equal to another ſuperficies, that is, 12 inches broad, and 12 inches long, for right againſt your breadth given, upon the other ſide of your Rule, you have the number of inches in length, which will make your breadth equal to a foot ſquare.

Example. Let the breadth given be 9 inches, right againſt 9 inches in the line of board meaſure, you find 16 inches, and ſo much in length will make one foot; by which length, meaſuring the whole length of your ſuperficies given, you may find the content thereof in feet. But if the breadth of your ſuperficies be leſſe than 6 inches, you muſt have recourſe to the Table, in the head whereof is ſet your breadth, and right under it the length of the ſuperficies which will make a foot of ſuperficial meaſure.

Example. Let the breadth given be 5 inches, then will 2 foot 4 inches, and 4 fifts of an inch in length make a foot.

Secondly, *Of the line of Timber-meaſure.*

IF a piece of Timber, or other ſolid be perfectly ſquare, ſeek upon your line the number of inches, that your ſolid will bear ſquare, and right againſt that number of inches, upon the other ſide is the number of inches and parts in length, which will make a foot.

Example. Suppoſe your ſolid given were 9 inches ſquare, that is, 9 inches on every ſide, right againſt 9, in the line of Timber meaſure, on the other ſide ſtandeth 21 inches, 3 eights of an inch almoſt, and ſo much in length will make a foot.

If it be but a ſmall piece of Timber which is to be meaſured, as under 9 inches ſquare, you muſt have recourſe to the Table, and ſeek the ſquare in the upper Rank of the Table, and right under, you have the feet inches, and parts that goeth to make a foot ſquare, as in the Table of board-meaſure.

Example. Let there be a piece of Timber propounded that is 5 inches ſquare, right under 5 inches in the head of

of the Table, I find 5 foot, 9 inches, 1 eight of an inch.

But if your ſolid given be not juſt ſquare, but broader at one ſide than at the other, then will this line or Table ſtand you in no ſtead, it being made for ſuch ſolids onely as are perfectly ſquare, which is but ſeldom, & therefore is not of much uſe: and the uſual way (by which the Carpenters doe meaſure ſuch Timber) is very erroneous, which is thus.

If a piece of Timber be 15 inches one way, and but 9 inches the other, they adde theſe two together, which make 24, and 12 the half thereof they take for the juſt ſquare of that piece, and therefore, ſay, that 12 inches or a foot in length will make a foot ſquare, but in this Example there will be 108 inches leſs then a true foot, for if you multiply 15 by 9, the product 135 is the baſe of the ſolid given, which being multiplied by 12 the ſuppoſed length of a foot, the product is but 1620 inches, whereas in a foot of Timber there muſt be 1728 inches 108 inches more.

To avoid this inconveniency, it is neceſſary that the Artificer ſhould have ſome Arithmetical skil, which being ſuppoſed he may *(by the 1 Prop. of the 8 Chap.)* meaſure any Board, and *by the 2 Prop. of the 10 Chap.* any piece of Timber, or other regular figure whatſoever, in which there will be ſome eaſe, if the inches be divided into 10 parts, but the beſt way is to divide the foot into 10 parts, and each of thoſe into 10 more.

The Tables by which the lines of Board and Timber meaſure are ſet upon the Carpenters Rule, may be computed by the following Problems.

PROBLEM I.

The breadth of a long ſquare being given in inch-meaſure, to find the length of a ſuperficial foot in the ſame meaſure.

AS the breadth given, is to 12 inches, or a foot in breadth; ſo is 12 inches or a foot in length, to the length required.

Or

Or thus:

As the breadth given, is to 144 inches the quantity of a superficial foot, so is 1 foot; to the length required. And therefore 144 inches being divided by the breadth given, the quotient shall be the length required.

Example. Let the breadth of a board be 5 inches, and you would know how much of that board in length will make a foot or 144 square inches: divide 144 by 5, the quotient is 28 $\frac{4}{5}$ inches, that is, 28 inches 3 quarters $\frac{1}{20}$, and so much in length will make a foot of board, for 28 $\frac{4}{5}$ inches being multiplied by 5 the product will be 144.

In like manner, if the breadth of a board given, be 13 inches, 9 $\frac{9}{14}$ inches in length will make a foot of board, and so of any other.

PROBLEM 2.

The breadth and depth of a piece of Timber being given, to find how much of that Timber in length, will make a foot.

MUltiply the breadth by the depth, and divide 1728 the number of cubic inches in a foot by the product, the quotient shall be the length required. For,

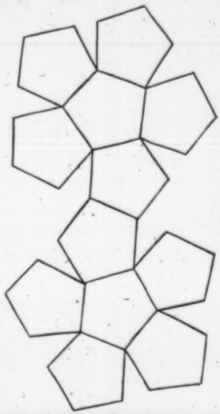
As the rectangle made of the breadth and depth, is to 1728, so is 1, to the length.

Example. Let the breadth of a piece of Timber be 17 inches and the depth 9, the product of these two is 153, now then if you divide 1728, by 153, the quotient will be 11.2941, that is, 11 inches $\frac{1}{4}$, and somewhat more.

2 *Example.* Let the breadth of a piece of Timber be 7 inches, and the depth as much, the product of 7 by 7 is 49: Now then if you divide 1728 by 49, the quotient is in inches 35.265 that is, 2 foot 11 inches $\frac{1}{4}$, and somewhat more, and so of any other.

CHAP.

Fig 45



Dodecahedron

Fig 48

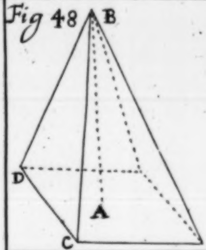


Fig:49

Fig 46

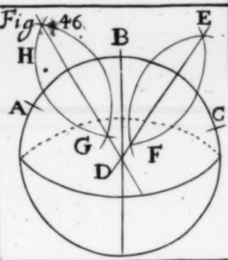


Fig:50

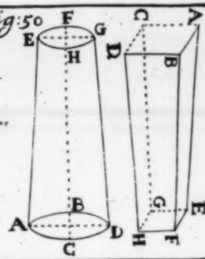


Fig 47

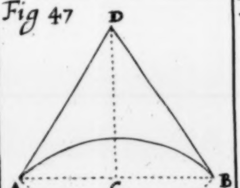


Fig:51

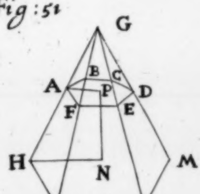
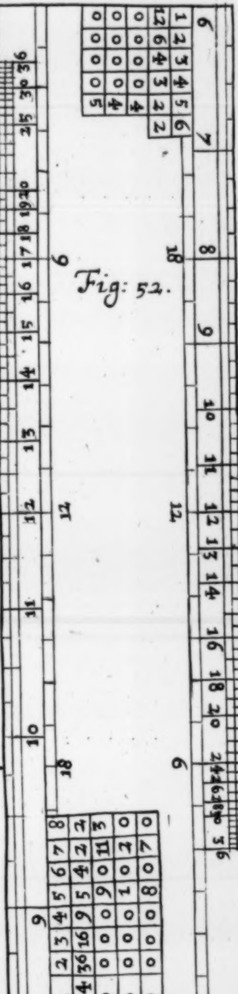


Fig: 52.



CHAP. XIII.

Of the Art of Navigation in General.

THere be four things upon which the Practise of Navigation is especially grounded, *viz.* 1 The Longitude. 2 Latitude. 3 Course. 4 Distance.

Touching the Longitude, though it may be found by the other three, yet hitherto there hath not been delivered any general Rule, true and practiseable, whereby the longitudes of places might be immediately and ordinarily found of themselves.

The Latitudes of places may be immediately found by observation of Sun and Stars, as shall be shewed in the 5 Chapter of the Description and use of the Globe.

The third thing to be considered in the Art of Navigation, is the Course or Line, by which the ship must goe. Now the Course of a ship upon the Sea, dependeth upon the Windes; and the designation of these depends upon the certain knowledge of one principal Wind; which considering the situation and condition of the whole Sphere: ought in nature to be *North*, or *South*: the *North* to us upon this side of the line, the *South* to those in the other Hemisphere; for in making this Observation, men were to intend themselves towards one or other certain fixed part of the Heavens, and therefore to the one of these.

In the *South* part there is not found any Star so notable, and of so neare distance from the Pole, as to make any precise or firm direction of that Wind: but in the *North* we have that of the second magnitude in the taile of the lesser Bear, making so small and inceaseable a Circle about the Pole, that it cometh all to eno as if it were the Pole it self.

This pointed out the *North-wind* to the Mariners of old especially, and was therefore called by some the

Lead or *Lead-star*, but this could be onely in the night, and not alwayes then.

It is now more constantly and surely shewed by the *Needle* touched with the *Magnet*, which is therefore called the *Lead* or *Load-stone*, for the same reason of the leading and directing there courses to the *North* and *South* parts of the Earth, not in all parts directly, because in following the constitution of the great *Magnet* of the whole Earth, it must needs be here and there led aside towards the *East* or *West* by the unequal temper of the Globe, consisting more of Water than of Earth, in some places, and of Earth more or less Magnetical in others.

This deviation of the *Needle* the Mariners call *North-easting*, and *North-westing*, as it falleth out to be, otherwise, and more artificially, the variation of the Compasse, which, though it pretend uncertainty, yet proveth to be one of the greatest helps, that Seamen have.

The *North* and *South* winds being thus assured by the motion either of Direction or Variation of the Needle the Mariner supposeth his ship to be (as it alwayes is) upon some Horizon or other, the Center whereof is the place of the ship represented by the Center of the Mariners Compass, the Diameter whereof we will suppose to be the line of *North* and *South*, which being crossed by another line through that Center at right angles, sheweth the *East* and *West*, and so they have the four Cardinal winds, crosse again each of of these lines, and they have the eight whole winds, as they call them. Another division of these maketh eight more, which they call half winds, a third maketh sixteen, which they call the quarter windes: and so in all there are thirty and two.

Every one of these winds is otherwise termed a several point of the Compass, and the whole line consisting of two winds, as the line of *North* and *South*, or that of *East* and *West*, is called the Rumb. The winds and Rhumbs thus assigned by an equal division of a great

great Circle into 32 parts, the angle which each Rumb maketh with the Meridian is easily known, for if you divide a Quadrant or 90 deg. into eight parts, you have the angles which the eight winds reckoned from *North* to *East*, or *West* doe make with the Meridians and those reckoned from *South* to the *East* or *West* are the same, and for your better direction are here exhibited in the Table following.

Here note, that the angle which any point of the Compasse maketh with the Meridian, is called the Rumb: and the angle which it maketh with any parallel, is called the Complement of the Rumb.

A *Table shewing the angles which every Rumb maketh with the Meridian.*

North	*South*	D	M	D	parts	*South*	*North*
		02	49	02	8125		
		05	37	05	6250		
		08	26	08	4375		
NE *by* E	S *by* E	11	15	11	2500	S*by*W	N *by*W
		14	04	14	0625		
		16	52	16	8750		
		19	41	19	6875		
NNE	SSE	22	30	22	5000	SSW	NNW
		25	19	25	3125		
		28	07	28	1250		
		30	56	30	9375		
N.E *by* N	SE *by* S	33	45	33	7500	SW*by* S	NW*b*N
		36	34	36	5625		
		39	22	39	3750		
		42	11	42	1875		
NE	SE	45	00	45	0000	SW	NW
		47	49	47	8125		
		50	37	50	6250		
		53	26	53	4375		
NF *by* E	SE *by* E	56	15	56	2500	W*b* W	NW*b*W
		59	04	59	0625		
		61	52	61	8650		
		64	41	64	6875		
ENE	ESE	67	30	67	5000	WSW	WNW
		70	19	70	3125		
		73	07	73	1250		
		75	56	75	9375		
E *by* N	E *by* S	78	45	78	7500	W *by* S	W *by* N
		81	34	81	5625		
		84	22	84	3750		
		87	11	87	1875		
East	*East*	90	00	90	0000	*West*	*West*

CHAP. XIV.

How to find the Azimuth of the Sun by the Scale of Versed Sines.

TO the line of Versed Sines, of which we have already shewed the construction, in the third Chapter. Mr. *Samuel Foster*, late Professor of Astronomy in *Gresham* Colledge *London*, hath very conveniently joyned the 12 signes of the Zodiack in the use whereof as there he hath explained himself the place of the Sun, is still supposed to be known, and is set to the line according to the Suns Declination from 90 both wayes, and therefore *Aries* and *Libra* having no Declination are set at 90 d. in this line of versed sines, and *Taurus* having North Declination 11 deg. 51 min. it set so many degrees and parts from 90 towards the beginning of the Scale, and *Scorpio* having South Declination as much, is in like manner placed so many degrees and parts from 90 towards the end of that line, and for the rest, according to their several Declinations: but how the Reader should find the Suns place for every day in the year, he did not think convenient to explain there, nor was it indeed any great omission, seeing it is set down in so many of our yearly *Almanacks*, yet in the print of that Scale which in this Treatise I doe recommend to the Reader, there is a line purposely added to supply that defect, in which you have the Suns place and Declination both, to every day of each moneth in the year, and may or may not be added to the Scale, as every one shall find it most convenient for his own use.

To find the Suns Azimuth by this line or Scale, three things must be given: the latitude of the place in which you are, 2 the Suns altitude above the Horizon, and the Suns place in the Zodiack: these three being given, find the sum and difference of the Complement of your latitude, and Complement of the Suns altitude: then having made A B equal to the length of the whole Scale, count upon the same Scale the sum and difference before found.

After this, take with your Compasses the distance from the Suns place to the sum, and setting one foot of that extent upon B, with the other describe the ark CD. So again, take the distance upon the Scale from the Suns place to the difference, and with that extent upon the Center A, describe the ark E F: which done, draw the straight line D E so, as it may just touch those arks, cutting the line A B in G: so shall B G (being measured upon the Scale from the beginning of it) shew the Azimuth from the South. And A G measured upon the same Scale will give the Azimuth from the North. *Fig.53.*

Example. August the 10[th] the Suns place is in *Scorpio* 3 deg. his altitude in the latitude of *London* 51 d. 53 m. is 14. 25 degrees.

Now then the Complement of latitude	38.47
Complement of Suns altitude	75.75
Their sum	114.22
Difference	37.38

Now then, having drawn the line A B equal to your whole Scale of versed sines, set one foot of your Compasses in 3 d. of *Scorpio*, and open the other to 114.22 d. in that Scale, and with that extent upon the Center B, describe the arch C D, then open your Compasses from 3 degrees of *Scorpio*, to 37.38 degr. in that Scale, and with that extent upon the point A, describe the arch E F, and draw the line D E. Then is B G being mea-

sured

sured upon that Scale 48.13, the Suns Azimuth from the South, and AG 141.87 the Suns Azimuth from the North.

CHAP. XV.

Of the Variation of the Compasse.

THat the course or way of a ship upon the Sea, is in the general directed by the Needle touched with the *Magnet*, hath been already shewed in the 13th Chapter: but because this doth not in all places directly point into the *North* and *South*, as hath been said: before we proceed any further, it is fit, to set down some directions for the more certain knowledge of those points, how much the direction of the Needle differeth from the true position of them, and by consequence from any other point in the whole Compass.

Now the *North* and *South* points cannot be more certainly known, then by the Azimuth of the Sun, or his amplitude from the *East* or *West* at the time of his rising or setting. Or by his Azimuth and distance from the *North* and *South* part of the Meridian at any other time of the day, as hath been shewed in the former Chapter, and shall be otherwise shewed when we come to speak of the use of the Globe: by either of which wayes the Azimuth of the Sun being found the direction or variation of the Compass is easily found also: for supposing the circumference or outermost edg of the Card or flie of the Compass to be divided into 360 degrees, and the point of the Needle to be placed directly under the hourline or *North* and *South* points, you are to observe at Sun-rising or setting, how many degrees the Sun is from the *East* or *West* of the Compass, which number of degrees, (if they agree with the amplitude

plitude found by Calculation) then hath the Compass no variation: but if they differ, look how many degrees the difference is, so much is the variation.

Example. Suppose the Suns amplitude to be 21 deg. 50 centesmes northerly: then I know that the Sun should set so much from the *West* towards the *North*: but observing with my Compass at Sunsetting suppose I find it to set but 17 degrees from the *West* point of my Compass to the Northward, therefore the Needle doth vary from the true *North* and *South* points 4 degrees 50 centesemes.

And to find which way that Compass varieth, you must observe whether the amplitude found, be towards the right or the left hand of the Sun-rising or setting; If it be on the right hand, the variation is Easterly, if on the left Westerly: for when a mans face is towards the *North*, the *East* is on his right hand, and the *West* on his left.

As in this Example, I find by the amplitude, that the Sun should set 21 deg. 50 parts, from the *West* part of my Compass Northerly, but setting the Sun, I see that this 21 degrees 50 parts of my Compass is more towards the right hand, then the place of Sun-set, therefore I conclude that the variation is Easterly of the *North*, and consequently that all the other points of my Compass direct more toward the right hand, then they should by 4 degrees and an half. And thus is the Needles variation to be found, by observing and comparing the Suns amplitude.

And in like manner may the variation be known by the Suns Azimuth.

Example. Admit the Suns true Azimuth from the *North* to be 107 deg. 50 centesmes, and 102 deg by the Compass, the difference between these 5 d. 50 centesms. is the present variation of the Compass.

But to know whether this variation be Easterly or Westerly, I consider that by the Suns true azimuth found by Calculation, the Sun should have been from the

the *North* 107 deg. 50 cent. that is, from the *East* point of the Compass to the Southward 17 deg. 50 centesm. whereas setting it with my Compass, is was from the *East* to the Southward but 12 degrees: so that the degree whereon the Sun should have been, was more toward the right hand, than the degree whereon it was: therefore I affirm the variation to be Easterly 5 degrees 50 centesmes.

CHAP. XVI.

Of dividing the Log-line, and how to reckon by it the way or distance that the ship is gone from any point.

THe finding of the distance of a ship to any place from which she hath departed, is the last of the four things propounded, as necessary in this Art of Navigation; and this is generally done by a Line fastened to a little Log of wood let down into the water, for by the length of the Line which this Log shall draw out of the ship in some certain quantity of time, is usually reckoned the distance that the ship hath run in that Line, or may run for some small portion of time after: and although it is not a Rule infallible, yet doth it many times stand the Mariner in good steed.

This way of finding the distance run, is grounded upon this opinion, that 5 of our feet doe make a pace, and that 1000 of such paces make a mile, and 60 such miles doe make a degree; and thus a degree should contein 300000 feet; but this errour hath been long since detected by our worthy Contry-man Mr. *Richard Norwood*, who by the experiments of others as well as

of

of his own, hath found it contein a greater number of feet, the measure of a degree assertained to us by his experience is 367.200 of our *English* feet: but for the rotundity of the numbers, and because it is safer to reckon of the least, rather than too much, he doth suppose a degree to contein onely 360.000 of our feet. And reteining still the same division of a degree into 10 miles or 20 leagues (as hath been formerly used) a mile will contein 60.00 feet.

Now supposing the time of the running out of the *Log-line* to be measured by an half-minute-glass, if we observe how many feet she runs in half a minute, we may thereby find her way for an hour or 4 hours, or for any time proposed.

As, admit that 50 foot of the *Log-line* should run out in half a minute or 30 seconds of time, it is apparent that then the ship runneth a mile, an hour, 100 feet being the 60[th] part of a mile: and therefore,

As 1 *minute, is to* 60 *minutes: So is* 100 *feet, to* 6000 *feet. Or, As* 1 *minute, is to* 60: *So is* 200 *feet, to* 12000 *feet or miles.* And so for any other.

Upon this ground, if at half a score fathoms or more from the Log, you make a mark, and beginning from thence measure 50 feet, you may there make the first knot, and 50 foot further two knots, and 50 foot further three knots, and so proceeding look how many knots there are run out in half a minnte, so many miles doth a ship run in an hours time, and every 5 foot more besides the knots, is a tenth part of a mile; as it there were run out 6 knots and 45 feet in half a minute the ships way is after the rate of 6 miles and 4 tenths of a mile in an hours time, &c.

Or, reteining the same number of feet, for the quantity of a degree, *viz.* 360.000, if you shall allow 100 miles to a degree, every such mile will contein 3.600 feet, 5 of which miles will be equal o 3 of the former, for 5 times 3600 is 18.000 feet, and 3 times 6000 is 18,000 feet also.

Now, if according to this proportion, the time of running out of the Log shall be measured by a Glass of 36 seconds, or the hundreth part of an hour, and shall make a knot at every 36 feet in the Log-line, look how many knots there are run out in that Line, so many miles doth a ship run in an hour, and every 36 feet being divided into 10 parts, that is, every 3 foot and 6 tenths of a foot, every of those divisions, is a tenth part of a mile more, and thus, if there were run out 18 knots and 3 tenths, the ships way is after the rate of 8 miles and three tenths of a mile in an hour. By either of these, the account of the ships way may be exactly enough kept, and the Mariner is left to his choice, though the latter (as being most easie and ready for use) deserves a better acceptance, then as yet it hath found.

Chap. XVII.

Of Sailing by the plain Chart.

Hitherto we have shewed how two of the four principal things in the Art of Navigation, *viz.* how the Rumb and Distance may each of them be found by themselves: we will now shew, not onely how serviceable these together with the latitude, are for the finding out of the longitude of places: which is the fourth thing desired; but also how any two of these four being given, the other may be found: and this may be done, either by straight lines, or by arches of great Circles.

Now, for the finding of these by straight lines, there are two wayes: the first supposeth the degrees of longitude in every parallel of the Æquinoctial, to be equal to the degrees thereof, as those in latitude are, and this is called, plain Sailing, or Sailing by the plain Chart.

The

The other ſuppoſeth the degree of longitude in every parallel to be equal to thoſe in the Æquinoctial as the other doth, but becauſe in truth they are not ſo, it therefore ſuppoſeth every degree of latitude to increaſe in ſuch proportion, as every parallel of longitude doth decreaſe from that in the Æquinoctial, and this is that which we call, Sayling by *Mercators* Chart: of both which ſo much hath been already written by others, that I ſhall not need to ſay much, I ſhall therefore onely ſhew by an Example or two in each, the uſe that plain Triangles have in this (no leſs famous than profitable) Art of Navigation.

Problems of Sayling by the plain Chart.

PROBLEM 1.

The longitude and latitude of the place from whence you came being known, with the Rumb you ſailed upon, and diſtance run, to find the longitude and latitude of the place to which you are come.

LEt the place from whence you have ſailed be A, whoſe latitude we ſuppoſe to be 52 and longitude 35 degrees, the Rumb upon which you have ſailed we ſuppoſe to be the third Rumb from the Meridian Eaſtward, that is, *S. E.* by *S.* 33 deg. 45 min. or in Decimal numbers 33.75 degr. and the diſtance which you have ſailed upon this Rumb, let be 96.214 leagues. Then in the Right angled plain Triangle A C E we have known, the angle C A E 33.75, and the Hypotenuſe A E 96.214, and the legs A C the difference of latitude, and C E the difference of longitude are required.

Now therefore if you protract the given angle CAE 33.75, upon the point A, as hath been ſhewed, and upon the line A E, ſet off the diſtance run 96.214 leagues, or rather 288.642 miles: if you would reckon by 60 m. to a degree, 3 of which miles are anſwerable to one

 league:

league: the difference of latitude A C will be 240 miles, that is, dividing them by 60, the number of minutes in one degree, 4 degrees just, and E C the difference of longitude will be 160.36 miles, that is, 2 degrees 40 minutes.

But if you make A E 96.214 leagues to be 481.07 miles, by allowing 5 miles to every league, as you ought to doe if you reckon 100 parts to one degree, the difference of latitude A C will be 400 miles, or 4 degrees, and the difference of longitude, C E 267.27 miles or 2 degrees, 67.27 parts of a degree, which is in effect the same with the former.

PROBEM 2.

The longitude and latitude of the place from whence you came, with the Rumb upon which you have sailed, and the latitude of the place to which you are come, being given to find the distance and difference of longitude.

LEt A represent the place from whence you set sail, whose longitude and latitude suppose to be as in the last Problem; and the Rumb upon which you have sailed the same also, *viz. S. E.* by *S.* 33.75 degrees, let E represent the place to which you are come, whose latitude (found by observation) suppose to be 4 deg. more Northward than the place at A. Then in the Right angled plain Triangle A C E, we have known the angle C A E 33.75, the leg A C 4 degr. the difference of latitude, to find the hypotenuse A E, and the difference of longitude C E.

To perform this, you must protract the angle of the Rumb C A E 33.75 as before, and draw the line A E, where it crosseth the line of longitude C E is the place to which you are come, and therefore a Ruler laid to E, so that it may cut like number of degrees at the top and bottom of your Chart, will give for the difference of longitude 2.67 degr. and A E being measured in the side of your Chart, or by a like Scale of equal parts

wil

will give you 481.07 such miles, whereof 100 doe make a degree.

Other Problemes of plain Sayling might here be added, but these we deem sufficient to illustrate that, which is well enough known to be egregiously false, and erroneous, and therefore omitting what might otherwise be said concerning this way of sayling, come we now to that, which is in it self much more exact, and may be performed almost with as much ease, the second way of sailing by strait lines, known by the name of *Mercators* Chart.

CHAP. XVIII.

Of Sayling by Mercators Chart.

THe true difference between this and the plain Chart, hath been declared in the former Chapter: in this we shall briefly shew how such a Chart may be made, and to prevent a tedious Description thereof, we shall for that refer the Reader to Mr. *Wrights* error of Navigation, or to what we have borrowed from him, in our Mathematical Institution. In this place it will be sufficient, if we set down the proportion onely which the Diameters, and Semidiameters that every parallel to the Æquinoctial hath to the Meridian.

And the Semidiameter of every parallel, Hath such proportion to the Semidiameter of the Meridian: As the Semidiameter of the Meridian; Hath to the hypotenuse (that is to the Secant) of the parallels latitude.

If therefore the degrees in every parallel be by supposition (as in this Chart they are,) made equal unto those in the Æquinoctial, the first degree of latitude from the Æquinoctial must be equal to the Secant of one degree, and the space from the first degree of latitude to

 the

the second, must be equal to the Secant of two degrees, and so forward.

Now then if it be required to project such a Chart for any particular Voyage, as suppose from the latitude of 52, to the latitude of 56 degrees, having drawn the line A B, and having crosseth it at right angles with another line A E representing the parallel of 52 degr. you must then take the Secant of 52 from your Scale, and set it from 52 to 53 on both sides of the Chart, and draw the parallel 53.53.

Again, take the Secant of 53 from your Scale and set it upon your Chart from 53 to 54, and so draw the parallel 54.54. And so you are to draw the rest of the parlallels.

Then for the Meridians, or Divisions of the line B C, they are all equal to the Radius, if therefore you take the Radius, and run it above and below; you shall make the spaces or distances of the Meridians such as in the bottom of the Chart are figured with 1,2,3,4,5.

These degrees thus set on the Chart, may be subdivided into equal parts, which in the graduations above and below ought to be so: but in the graduations upon the sides of the Chart, they ought, as they grow higher, still to grow greater; and yet the difference is so small, that it cannot produce any considerable errour, though the subdivisions be all equal. Divide them therefore, either into 60 min or miles, or into 100 parts or miles, which is indeed the best; the use of your Chart thus drawn shall be explain'd in the following Problems.

PROBLEM I.

The longitude and latitude of the place from whence you came, with the Rumb upon which you have sailed, and distance run being given, to find the longitude and latitude of the place to which you are come.

LEt A represent the place from whence you come, whose latitude suppose to be 52 deg. North, and longitude 35 degrees, the Rumb upon which you have sailed

sailed we suppose to be the third Rumb from the Meridian Eastward, that is, *S. E.* by *S.* 33 deg. 45 min. or in Decimal numbers 33.75 degr. and the distance sailed upon the Rumb, let be *96.214* leagues. Then in the Right angled plain Triangle A B C we have known B A C 33.75 the angle of the Rumb, the hypotenuse A C *96.214* leagues, to find A B the difference of latitude, and B C the difference of longitude.

The angle of the Rumb must be protracted upon the point A, as hath been shewed in the plain Chart: but the distance run *96.214* leagues, or rather 481.07 such miles whereof 100 make a degree, must be taken from the Meridian line, at the latitude from whence you come, *viz.* 52 deg. upwards, that is, 4.81 degrees, and set from A to C, so shall the point C represent the place to which you are come: upon which point having drawn B C parallel to the latitude of 52 degrees, A B your difference of latitude will be found to be 400 miles or 4 degrees, and B C your difference of longitude 455 miles or 4.55 deg. 1.87 deg. more than the plain Chart.

PROBLEM 2.

The longitude and latitude of the place from whence you came with the Rumb upon which you sailed, and the latitude of the place to which you are come being given, to find the distance and difference of longitude.

LEt A represent the place from whence you come, whose longitude and latitude suppose to be as before and the Rumb also, *viz. S. E.* by *S.* 33.75 degrees. Let C represent the place to which you are come, whose latitude found by observation, suppose to be 4 degrees more Northward than the place at A. Then in the Right angled plain Triangle A B C, we have given the angle B A C 33.75 the angle of the third Rumb from the Meridian, and the leg A B 4 degrees of latitude more Northward than the place at A. To find the difference of longitude B C, and the distance run A C:

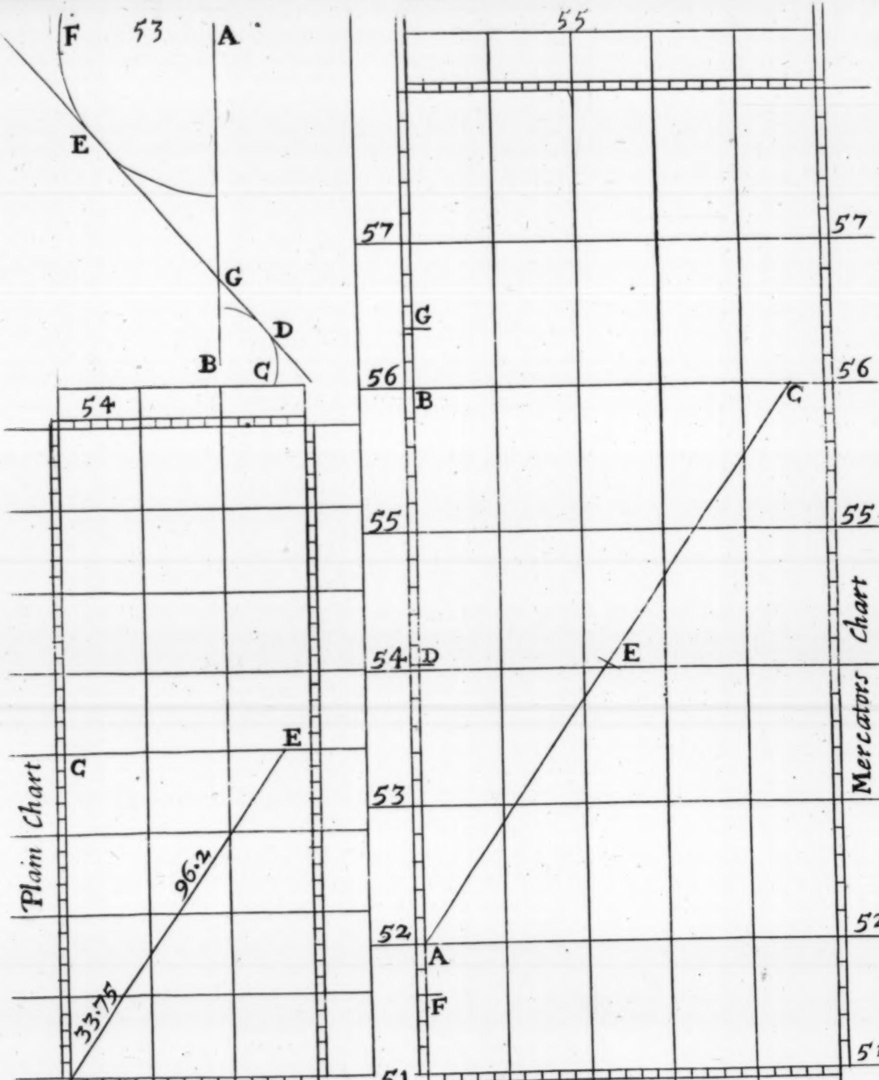
and

and both these are plainly exprest upon the Chart, the difference of longitude B C is by inspection degr. 4.55 parts. And A C the distance run may thus be measured, divide the space that is in the Meridian line between the two latitudes at A and B into two equal parts, which in this Example will fall in the point D, then take with your Compasses half the length of the line A C, that is the length of A E or E C, and setting one foot of that extent in the point D, which is the middle point between the two latitudes, you shall find that the other point set downward, will reach to F, that is, to 51.54, then keeping the one foot still fixed in the point D, turn the other upward, and it will reach to G, that is, 56.71: and therefore the degrees in the Meridian line between these two points F and G, are 4.81 deg. or 481 miles.

By these few Problems, the use of the Plain or of *Mercators* Chart, is (as we conceive) sufficiently explain'd, he that desires more ample instructions, may read what Mr. *Gunter*, Mr. *Foster*, Mr. *Norwood*, Mr. *Philips*, and Mr. *Collins* have written of this subject, in whose works you may be abundantly furnished with variety of questions, which being wel considered, a little Practise will render the things, to be very plain and easie, which at the first are hard and difficult to be conceived.

FINIS.

Plain | Chart


F
53
A
E
G
D
B
C
54
C
E
A
33.75
96.2
Plain Chart
55
57
G
56
B
C
55
54
D
E
53
52
A
F
51
Mercators Chart

THE DESCRIPTION *AND* USE OF THE TERRESTRIAL AND COELESTIAL GLOBES.

By JOHN NEWTON, *M. A.*

LONDON,
Printed for Robert Horn, *at the* Turks-head
in Corn-hill, *neer the* Royal *Exchange.*

Anno 1660.

Fig : 1

Fig : 2

page 91

THE DESCRIPTION and USE of the GLOBES.

CHAP. I.
What a GLOBE *is.*

A *Sphere* or *Globe* is an artificial representation of the Heavens, or the Earth and Waters, under that form & figure of roundness which they are supposed to have: shewing in a just proportion and distance, every particular constellation in the Heavens, and each several Region and tract of ground in the earth.

This description or representation is by circles, great and small, some of which are expressed upon, and others are framed without the Globe. The circles without the Globe are chiefly two: the Meridian, and the Horizon, the one of brass, the other of wood. Circles indeed they are not, to speak properly, because in a strict sense no line is supposed to have any breadth, whereas both these have breadth allowed them, that

such things might be written upon them, as might render them the more useful in all positions of the Globe, and therefore they being of a circular form, notwithstanding the impropriety of speech, use will have it so, and we must call them the *Meridian* and *Horizontal Circles*.

CHAP. II.

Of the Meridian without the Globe.

THe brass Meridian is divided into 4 equal parts or Quadrants, and each of them are subdivided into 90 deg. that is, 360 for the whole circle. The reason why this circle is not divided into 360 degrees throughout, but still stoping at the 90th, beginneth again with 10, 20, 30 &c. is, because the uses of this Meridian, so far as in degrees they are concerned: require no more than that number.

As for Example: One use of the *Meridian* is to shew the *Elevation* of the *Pole*, but the Pole cannot be elevated above 90 degrees. Another is to shew the *Latitude* or distance of a place from the Æquator, which also can never exceed the 4th part of the circle; for no place can be further distant from the Æquator than the Pole, which is just that number of 90 degrees.

CHAP. III.

Of the Axis and Poles of the Globe, and of the Hour-circle.

FRom the *North* and *South* ends of this Meridian a strong Wyer, or Brass, or Iron is drawn, or supposed to be drawn (for the Artificers do not alwayes draw it quite through) by the center of the Globe representing the Axis of the Earth. The

North

North end whereof ſtandeth for the *North*, the *South* end for the *South* Pole of the earth. Upon the *North* end, a ſmall circle of braſs is ſet, and divided into two equal parts, and each of them into twelve, that is, twenty four in all. This circle is the onely one above the Globe, which is not imagined upon the Earth, but is there placed to ſhew the hour of the day and night, in any place where the day and night exceed not 24 hours, therefore it is called *Cyclus Horarius*, the Hour-circle; for which purpoſe it hath a little braſs pin, turning about upon the Pole, and pointing to the ſeveral hours, which therefore is called, the *Index Horarius*.

The ſmal circle is framed upon this ground, that in the Diurnal Motion of the Heavens, 15 degrees of the Æquinoctial riſe up in the ſpace of each hour, that is, 360 degrees, or the whole circle, in the ſpace of 24. So that the *Cyclus Horarius* is to be framed to that compaſs, as that every twenty fourth part of it, or one hour, is to bear proportion to 15 degrees of the Æquator below it. And ſo in turning the Globe about, one may perceive, that while the pin is moved from any one hour to another, juſt 15 degrees of the Æquinoctial will riſe up above the Horizon upon one ſide, and as many more goe down below it on the other.

CHAP. IV.

Of the Horizon.

THe other Great Circle without the Globe is the Horizon, upon which (yet not as due to this circle, more than any other, but becauſe there is more room) the *Geographers* ſet down the 12 Signes with their names and Characters.

And becauſe every ſign of the Zodiack conteineth 30 degrees, which is 360 for the whole circle, the Hori-

zon is divided into 360 degrees indeed as it ought, but not from 10, 20, 30, 40, and so throughout, but by thirties, that is, 10, 20, 30, and 10, 20, 30, and so along, to make the division conform to the 12 Signes, to each of which, as I said, is alotted the number of 30 degrees. And the reason is in reference to the Suns Annual Motion, in the course whereof he dispatcheth every day one degree under or over. So that he passeth through each of the signes in, or much about, the space of 30 dayes. So that, though some of the 12 moneths, answering to the 12 signes, consist of one day more than 30, and one of 2 dayes less, yet take them one with another, and the dayes of every moneth correspondent to the several degrees of every sign, or without any considerable difference. And after that rate or much about it, they are placed upon the Horizon, to shew in what degree of any sign the Sun is every day of the year. And to this purpose, there is set down upon the same Horizon a Kalender, and that of three sorts in some Globes, of two in the most: One whereof is called the *Julian*, or Old, the other, the *Gregorian*, or New Account, reckoning this latter 10 dayes before the former, and the third sort where it is found 13. The ground of difference dependeth upon the Suns motions, which must be known by Calculation, not by the Globe, and so not apperteining to our present purpose.

Of these two circles there needeth not more to be said at present, onely we may observe, that it was ingeniously devised of those who first thought upon it to set one Meridian and one Horizon without the Globe, to avoid the confusion if not the impossibility of drawing a several Meridian, and a several Horizon for every place, which must have been done, if this or the like device had not been thought upon.

CHAP.

CHAP. V.

Of the Quadrant of Altitude, and the Compass.

BEsides the circles framed without the Globe, there are two other appendants: the one relating to the Meridian, the other to the Horizon.

The first is the Quadrant of altitude, and is a thin brass plate representing the fourth part of a great circle, and so divided into 90 degrees, called therefore the Quadrant; and the Quadrant of altitude, because it measureth the heigth of the Stars upon the Cœlestial Globe, to which it doth most properly belong. The business it hath to do in *Geography*, is to set out the Zenith of any place, and consequently to shew the angle of position, or bearing of one place to another: and is therefore affixed to the Meridian with a little Screw-pin, to be removed at pleasure from the Vertical point of any place to the Vertical point of any other.

The second appendent is the Compass, which is a Needle touched with a Load-stone, and set in a Box upon the foot of the Horizon, upon the *South* side, such another as we see in ordinary Pocket-Dials for the Sun. The use of it here (as in those) is to point out the North and South, for the rectification of the Globe, as shall be more plainly said hereafter,

CHAP. VI.

Of the great Circles upon the Globe, and first of the Equator.

BEsides the Meridian and Horizontal circles without the Globe, there are two other circles drawn upon the Globe it self, which are common to the *Terrestrial* and *Cœlestial* Globes both, the one is called Equator, or the Equinoctial circle, this is the circle

circle drawn in the middle between the two Poles graduated throughout, and plainly dividing the Globe into two equal parts, from *North* to *South*, this is the circle of longitude in the *Terrestrial Globe*, that is, the longitude of places, are from such or such a Meridian alwayes reckoned in the *Equator*.

Crossing this circle obliquely in the middle is the *Zodiack*, the uttermost extent whereof towards the *North*, noteth out the Tropick of *Cancer*; towards the *South*, the Tropick of *Capricorn*, each of them distant from the *Equator* 23 degrees and a half, or not much more, as may be accounted in the great Meridian.

These two circles of the *Equator* and *Zodiack* are crossed by two other great circles also, which are called *Colures*: both which are drawn through the Poles of the World, and cut the *Equator* at right angles. The one of them passing through the intersections of the Equinoctial points, and is called, the *Equinoctial Colure*. The other passing through the points of the greatest distance of the *Zodiack* from the *Equator*, and is therefore called the *Solsticial Colure*.

Parallel to the Tropicks and at the same distance from the Poles as they are from the *Equator*, are set down upon the *Terrestrial Globe* the *Artick* and *Antartick Circles*, offering themselves to sight by their names, and distinction of breadth and colour, being represented by more full lines then some other circles equidistantly removed from the *Equator* at 10 degrees difference, and may be called the *Unnamed Parallels*.

The other great circles painted upon the *Terrestrial Globe* are the Meridians: where we must not think much to hear of the Meridians again. That of Brass without the Globe, is to serve all turns, and the Globe is framed to apply it self thereto. The Meridians upon the Globe will easily be perceived to be of a new and another use.

They are either the great or the lesse; not that the great are greater than the less, for they have all one

and

and the ſame center, and equally paſs through th Poles of the Earth: but thoſe which are called leſs, are of leſs uſe than that, which is called the great, though it be no greater than the reſt.

The great is otherwiſe called the Fixt and Firſt Meridian, to which the leſs are ſecond, and reſpectively moveable. The great Meridian is (as it were) the *Landmark* of the whole Sphere, from whence the longitude of the Earth, or any part thereof is accounted. And it is the onely circle which paſſing through the Poles, is graduated or divided into degrees; not the whole circle, but the one half, becauſe the longitude is to be reckoned round about the Earth. This great Meridian might have been planted in any place, as at *London*, *Oxford*, *Cambridg*, or any other: but muſt of neceſſity be ſet in one certain place of the Globe or other, as it is in every ſeveral Globe, though not in the ſame place in all. It were indeed to be wiſhed, nay, it will hardly be well with *Geography* until it be brought to the *Fortunate Ilands* where the *Greek Geographers* placed it at the firſt, not promiſcuouſly, but on one certain Iland among the ſeven: that of *Teneriff* hath by many been thought the fitteſt, and *Pico de Teide*, or *El. Pico the Peak* a mountain ſo called from the ſharpneſs of the top, the beſt and moſt convenient place there.

The leſſer Meridians are thoſe Black circles, which you ſee to paſs through the Poles, and ſucceeding the great at 10 and 10 degrees as in moſt Globes; or as in ſome, at 15 and 15 degrees difference. Every place, never ſo little more *Eaſt* or *Weſt* then another, hath a ſeveral Meridian. *Black-wal* hath a diſtinct meridian from St. *Pauls*, *London*, becauſe more *Eaſt*. *Weſtminſter* Abbey hath not the ſame, as neer as it is, for that it lieth *Weſt* from St. *Pauls*: The exact meridian whereof, muſt paſs directly through the middle; yet becauſe of the huge diſtance of the Earth from the Heavens, all theſe places, and places much further off may be ſaid to have the ſame meridian. And indeed

there is no very sensible difference, in less than a degrees difference of longitude, upon which ground the *Geographers* as well as the *Astronomers* allow a new meridian to every other degree of the *Equator*, which would be 180 in all, but except the Globes were made of an extream and an unusual Diameter, so many would stand too thick for the Description. Therefore most commonly they put down but 18, that is, at 10 degrees distance one from the other, the special use of the lesser meridians being to make a quicker dispatch in the account of the longitudes.

Some others set down but 12, at 15 degr. difference, aiming at this: That the meridians might be distant one from the other, a full part of time, or an hour; for seeing that the Sun is carried 15 degrees of the Equinoctial every hour, the meridians set at that distance must make an hour difference in the Rising or Setting of the Sun, to the several places, as if the Sun Rise at such an hour, such a day of the year at *London*; In a place 15 degrees more distant towards the *East* the Sun riseth an hour sooner, in a place 15 degrees towards the *West* an hour later, the same day of this, or that year.

CHAP. VII.

Of the distribution of the Earth, by Zones, Climes, *and* Parallels.

THe Globe of the Earth hath anciently been divided or distinguished into parts, three wayes, *viz.* by *Zones*, *Climes*, and *Parallels.*

By *Zones* or *Girdles*, it hath been divided by the four lesser circles into 5 parts.

The first *Zone* is that part of the *Terrestrial Globe*, which is comprehended between the Tropicks, in the middle where of is drawn the Equinoctial circle.

The

The second between the Tropick of *Cancer* and the the *Arctick Circle*.

The third between the Tropick of *Capricornus* and the *Anctartick Circle*.

The fourth is included in the *Arctick Circle* in the *North*.

The fifth in the *Antartick Circle* in the *South*.

By the supposed temperature of these *Zones*, the first, or that which is comprehended between the Tropicks, the Ancients called the *Torrid Zone*, and not to be habitable by reason of heat. Those towards either Pole, they called the *Frigid Zones*, and not habitable by reason of cold: But those comprehended between the Tropicks and the *Artick* and *Antartick Circles*, they call *Temperate*: but experience hath now taught us to know that there are inhabitants as well in the *Frigid* and *Torrid Zones*, as in these that are more temperate.

The Ancients did also distinguish the inhabitants of the *Zones*, from the diversity of shadowes of bodies into three sorts, viz. *Periscii*, *Heteroscii*, and *Amphiscii*.

The Inhabitants of the *Frigid Zones* (if any such were) they are termed *Periscii*; because the shadowes of bodies have there a circular motion, in 24 hours; the Sun neither rising nor setting but in a greater portion of time.

The Inhabitants of the *Temperate Zones* they called *Heteroscii*, because the meridian shadowes in one part of the world bend toward either Pole; towards the *North* among those which dwell within the Tropick of *Cancer* and *Antirk Circle*, toward the *South* among those which live within the Tropick of *Capricorn* and *Antartick Circle*.

The Inhabitants of the *Torrid Zone*, between the Tropicks, they called *Amphiscii*, because the meridian shadow according to the time of the year, doth sometimes fall towards the *North*, sometimes towards the *South*, when the Sun is in Northern signes, it

falleth towards the *South*, and towards the *North* when in Southern.

And because of the different site of opposite habitation, the Ancients have divided the Inhabitants of the Earth into *Periaci*, *Antaci*, and *Antipodes*.

The *Periaci* are those that live under the same meridian, and the same parallel also, being equally distant from the *Equator*; but in two opposite points of the same parallel.

The *Antaci* are such as have the same meridian, but live in divers parallels, yet equally distant from the *Equator*, though in divers parts.

The *Antipodes* are such as inhabite under one meridian but under two divers parallels, which are equally distant from the *Equator*, and in opposite points of the same, or such as inhabite two places of the Earth, which are Diametrically opposite.

The second and third wayes by which the Ancients did divide the Globe of the Earth into parts, viz. *Climes* and *Parallels*, are most what but one, differing not so much in nature as in quantity.

A *Climate* they define to be a space of Earth comprehended betwixt any two places, whose longest day differ in quantity half an hour.

A *Parallel* they define to be a space of the Earth comprehended between any two places, whose longest day differ in quantity but a quarter of an hour, so that every *Climate* conteineth two parallels. These *Climates* and *Parallels*, though they have equal difference, in respect of the dayes length, yet are they not equal in respect of the quantity of the Earth or Globe that constitutes the *Clime* or *Parallel*.

The first *Clime*, as also the *Parallel*, beginning from the *Equator*, is larger than the second, and the second is likewise greater than the third. Herein onely they all agree, that they differ equally in the quantity of the longest day. The Ancients reckoned but seven *Climates* at the first, to which number were afterward added

added two more; so that in the first of these numbers were comprehended 14 parallels, but in the latter 18.

In the meridian of some material Globes, there are described 9 parallels, differing from each other by the quantity of half an hour; after these there are others also set according to the difference of an whole hour, and last of all, those that differ a whole moneth are continued to the very Pole, each of them expressed in their several latitudes.

CHAP. VIII.

Of the Geographical Description of the Terrestrial Globe.

Hitherto we have shewed how the Globe of the Earth is both by the Ancient and Modern Writers described by circles: we will now give you another short Description of it, by such lines as are real and natural, the former being but imaginary; and serve onely to inform the fancy; in the former Description of this Globe by circles, we have considered the Earth and Water both together, because they both make but one Globe: but now we shall consider them a part: and first, the Earth or Land hath for many years past been Geographically divided into 4 parts, *Europe*, *Asia*, *Africa* & *America*.

Europe is bounded on the *North* with the Northern *Ocean*, and on the *South* with the *Mediterranean Sea*; On the *East* with the Floud *Tanais*, and on the *West* with the Western *Ocean*, and doth contein these Provinces.

Germany	*Denmark*	*Polonia*	*Englad*
Italy	*Norway*	*Hungaria*	*Scotland*
France	*Swedeland*	*Sclavonia*	*Ireland*
Spain	*Moscovia*	*Grecia*	*Sicilia*

Candia,

Candia, *Corsica*, *Sardigna*, *Nigropont*, are the chief and principal Ilands belonging to it.

Asia is bounded on the *North* with the *North Ocean*, on the *South* with the *Red Sea*, and the Gulf adjoyning on the *East* with the *East-Indian Ocean*, and on the *West* with the Floud *Tanais*, whose principal Regions are.

Anatolia	*Media*	*Parthia*
Syria	*Assyria*	*Tartaria*
Palestina	*Mesopotamia*	*China*
Armenia	*Chaldea*	*India*, and the
Arabia	*Persia*	Ilands thereof.

Africa is bounded on the *East* with the *Red Sea*, on the *West* with the *Atlantick Ocean*, on the *South* with the Southern *Ocean*, and on the *North* with the *Mediterranean Sea*.

The Provinces are, *Egypt*, *Barbaria*, *Ethiopia*, *Nubia*, *Abasmus*, *Alonometopa*.

The Ilands, *Madagascar*, or St. *Laurence*, St. *Thomas*, *Insula de Cape Verde*, *Insula de Canaria*, *& de Madinea*.

America by later observations is found to be bounded on the *East* with the *Atlantick Ocean*, on the *West* with the *West-Indian Ocean*, on the *North* with the Northern *Ocean*, and on the *South* with the *Magellanick* Sea, consisting of two parts, viz. *Mexicana*, and *Peruana*.

The Provinces of *Mexicana* are, *Nova Hispania*, *Terra Florida*, *Nova Albion*, *California*, *Norumbega*, *Nova Francia*, *Estotiland*.

The Provinces of *Peruana* are *Brasilia*, *Tisnada*, *Caribana*, *Catagena*, *Peru*, *Charas*, *Chila*, *Chica*, *Patagones*.

The chief Ilands of *Mexicana*, are: *Green-land*, *Island*, *Friesland*.

The chief Ilands of *Peruana*, are: *Hispaniola*, *Cuba*, *cum multis aliis in India Occi*, *Insula Margarita*. *Molucq Insula Remores*, *Java Major*, *Java Minor*, *Solomonis Insula*. And the other Ilands in *India Orientalis*.

He

He that desires a more ample description of the Terrestrial Globe Geographically, may consult Dr. *Heylins* Geography, this we deem sufficient for our present purpose, which is to inform the Reader in what part of the Globe to seek the quarter of the World he looketh for; and yet before we break off this discourse, it will not be amiss, to set down the proportion which the circumference of the Earth hath to the Heavens, and although this perhaps is not as yet certainly known, yet here to insert the several opinions of the Ancients will rather perplex the Reader with doubts, than prove the truth of the thing, we shall therefore content our selves with that proportion which a Country-man of our own, *viz.* Mr. *Richard Norwood* hath not many years since set down from his own experience in his *Seamans Practice*, which is, that one degree in the Heavens is answerable to 367200 of our *English* feet, but for the rotundity of the number, and being willing to account something too little, rather than any thing too much (because in Sea Voyages for which chiefly he took this paines, it is better to fall somewhat short of desiring Port, than to out run it) he accounteth but 360.000 *English* feet unto a degree, and this he divideth by 60, the number of minutes usually accounted in a degree or hour, then the quotient giveth 60 miles, for the quantity of a degree every mile conteining 60.00 feet: the like exactness will be found in allowing 100 minutes or parts to a degree or hour, making in each mile 3600 feet, and in the Art of Navigation will be somewhat more ready.

We come now to the description of the Watery part of this Globe, or rather the description of the Mariners Course upon the Waters, which is to shew the way of a ship, upon the Sea, and this dependeth upon the winds. The Designation of these, upon a certain knowledg of one principal, which considering the situation and condition of the whole Sphere, ought in nature to be *North*

or

or *South*. The *North* to us upon this side of the line, the *South* to those in the other Hemisphere, for in making this observation, men were to intend themselves towards one fixed part of the Heavens, or other; and therefore to the one of these. In the *South* part there is not found any Star so notable, and of so near a distant from the Pole, as to make any precise or firm direction of that wind. But in the *North* we have that of the second magnitude in the Tail of the lesser Bear, making so small and for the motion so insensible a circle about the Pole, that it cometh all to one as if it were the Pole it self.

This pointed out the *North wind* to the Mariners of old especially, and was therefore called by some the *Load* or *Lead-star*, but this could be onely in the night, and not alwayes then.

It is now more constantly and surely shewed by the *Needle* touched with the *Magnet*, which is therefore called the *Lead* or *Load-stone*, for the same reason of the leading and directing their courses.

The *North* and *South* winds thus assured by the motion either of Direction or Variation of the Needle, the Mariner supposeth his ship to be (as it alwayes is) upon some Horizon or other, the Center whereof is that of the ship.

The line of *North* & *South*, found out by the Needle, a line crossing this at right angles sheweth the *East* and *West*, and so they have the four Cardinal winds. Crosse again each of these lines, and they have the eight whole winds, another division of these maketh eight more, which they call half winds, a third maketh sixteen, which they call the quarter windes: so they are 32 in all.

The Compass therefore is an Horizontal Division of the 32 winds, upon a round piece of pastboard set in a box, in the Center whereof upon a pin, the Needle or Wyers first toucheth with the Stone are placed. These Compasses are represented as they may

upon

upon the Globe, by those Circles which you see divided into 32 parts, with their *Fleur de Lis*, alwayes pointing to the *North*. And though the winds are not set down by their names, yet they may be fetched from the Horizon without the Globe. And the Rumbs are drawn out at length circularly, if the course be upon a Meridian, *Equator*, or other parallel, otherwise they are Helispherical lines, as they call them, that is, partly Circular, and partly Helical, or Spiral, as you may discerne them to be described upon the Globle.

CHAP. IX.

Of the Circles which are proper unto the Cœlestial Globe.

IN the Cœlestial and Terrestrial Globes, the Meridian, Horizon, Æquator, Zodiack or Ecliptick rather, and the hour-circle are common: but the lesser Meridians which are described upon the Globe it self upon the Poles of the Equator, and serve to number the longitude of places by, are proper onely to the Globe of the Earth; and instead of these upon the Globe Cœlestial, there are described 12 Semicircles, from one Pole of the Zodiack unto the other, and passing through the beginning of the 12 Signes, and doe make six great Circles.

The first passeth the head of *Aries*, and beginning of *Libra*. The second through the head of *Taurus*, and the beginning of *Scorpio*: the rest in like manner doe follow in order, dividing the surface of the Globe into 12 equal parts, which are widest in the Ecliptick, and lessen themselves by little and little as they approach towards the Poles of the Zodiack, and at length doe wholy terminate therein, as the lesser

meridians in the Terrestrial Globe doe in the Poles of the Equator, and serve to number the longitude of the Stars by, alwayes reckoned from the beginning of *Aries*.

The entire superficies of any one of these parts, derives his name from the Signe, comprehended in the Ecliptick between each Semicircle; thus the Superficies lying between the two Semicircles drawn through the beginning of *Aries* and *Taurus* comprehending the Sign of *Aries* in the Ecliptick, is also called the sign of *Aries*: and all the Stars and Planets, and other points of Heaven between these two Semicircles on both sides of the Ecliptick to the Poles, are said to be in *Aries*, and so of the rest.

The Regions of the Earth in the Terrestrial Globe are placed according to their longitude, by the degrees of the Equator, and according to their latitude by the degrees of the Meridians from the Equator to the Poles thereof: So in like manner are the Stars placed in the Cœlestial Globe in their proper longitude, according to the degrees of the Ecliptick, and latitude according to the degrees of the circles of longitude from the Ecliptick towards it Poles: this latitude of the Stars is twofold *North* and *South*, *North* in those Stars which goe from the Ecliptick towards the *North* Pole, *South* in those which incline to the *South* Pole.

CHAP. X.

Of the difference and denomination of the Stars.

THe whole number of Stars hath been divided, by the Ancient *Astronomers*, who first applied themselves to the diligent observing of them, into two kinds. The first is of the Planets, or wandring Stars: the other of the fixed. The first of which

which they therefore called Planets or Wanderers, because they observe no constant distance or situation neither in respect of each other, nor in respect of those that are called fixed Stars. And these were so called, because they were observed alwayes to keep the same situation and distance from one another.

The Planets are in number seven, *Saturn*, *Jupiter*, *Mars*, *Sol*, *Venus*. *Mercury*, *Luna*, all which besides the Diurnal motion, by which they are carried about from *East* to *West*, by the Rapture of the first moveable, have also a free proper motion of their own: which they finish from *West* to *East*, according to the succession of the Signs upon the Poles of the Zodiack, each of them in a several manner and space of time: but because these are not expressed upon the Globe, we shall for a farther knowledge of these refer the Reader to our Treatise of *Astronomy*, and proceed to those which are called fixed Stars.

And of these the Ancients have left in their writings the number of one thousand twenty and two Stars, observed by them, in the Northern and Southern Hemispheres of Heaven: and that they might be the better discerned from one another, they have reduced them into fourty and eight Images or Constellations, distinguishing the Stars in every Constellation by several and distinct names, the names of the Constellations and number of Stars in each of them, are as followeth.

The Zodiacal Constellations are 12, viz.

Aries	14	*Cancer*	9	*Libra*	8	*Capricorn*	28
Taurus	22	*Leo*	27	*Scorpio*	21	*Aquarius*	42
Gemini	18	*Virgo*	26	*Sagittarius*	31	*Pisces*	34

The Northern Constellations are 21, viz.

Ursa Minor	7	*Lyra*	10	*Sagitta*	5
Ursa Major	27	*Avis*	17	*Aquila*	9
Draco	31	*Cassiopeia*	13	*Delphinus*	10
Cepheus	11	*Perseus*	26	*Equisectio*	4
Arctophilax	21	*Heniocus*	14	*Pegasus*	20
Corona Borealis	9	*Serpentarius*	24	*Andromeda*	23
Engonasus.	29	*Serpens*	18	*Triangulus*	4

The Southern Constellations are 15, viz.

Cetus	22	*Procyon*	2	*Centaurus*	37
Orion	38	*Argo*	41	*Fera*	19
Eridanus	34	*Hydra*	25	*Ara*	07
Lupus	12	*Crater*	7	*Corona Aust.*	13
Canis Major	18	*Corvus*	7	*Pisces Aust.*	11

As for the particular Names of the fixed Stars in each of these Constellations, we refer you to Mr. *Fosters* Catalogue, late Professor of Astronomy in *Gresham* Colledg, *London*, in which you will also find their true longitude and latitude, as they ought to be now described upon the Globe it self.

CHAP. XI.

Of the distinctions and affections of Spherical lines or arches.

Hitherto we have spoken of the Globe it self, together with its dimensions, circles, & other Instruments necessarily belonging thereunto. It remaineth now that we come to the practise of it, and declare its several uses.

And

Fig: 3

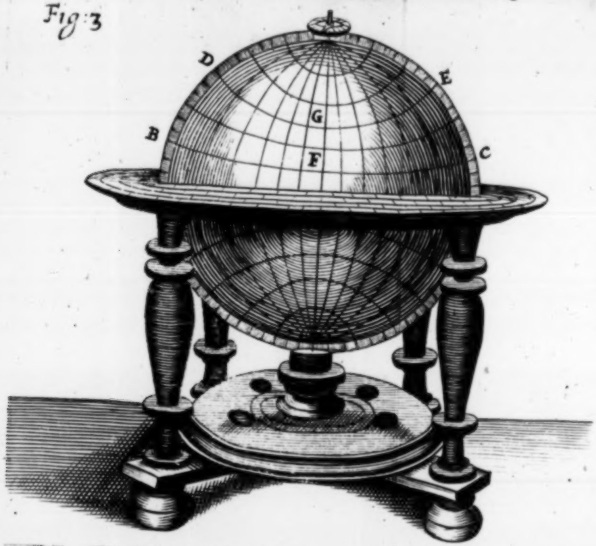


Fig: 4

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And the uses of the Globe, as to practise, are either such as concern the Heavens or the Earth, in either of which, if we should descend unto particulars, the uses would be more in number, than a short Treatise will contein: seeing therefore that all Problemes which concern the Globe, may be best, and most accurately resolved by the Doctrine of Spherical Triangles, we will contract those uses of the Globe, which otherwise might prove infinite, to such Problems as come within the compass of the 28 Cases of Right and Oblique angled Spherical Triangles.

And that the nature of Spherical Triangles may be the better understood, and by which of the 28 Cases the particular problems may be best resolved.

1 First, we will set down and that briefly, some general Definitions and affections which doe belong to such lines or arches of which the Triangle must be formed.

2 We will describe the kinds, parts, and affections of those Triangles, and how the things given & inquired in them, may generally be represented, and resolved upon and by the Globe: with the number of several Problems, that every Single, Right, and Oblique angled Spherical Triangle will afford.

3 We will exhibite a Catalogue of the most general and useful things which are inquired for, both in *Astronomy* and *Navigation*, and shew the several wayes by which they are, or may be found.

First, then of the Definitions and affections of those lines or arches of which the Triangle must be formed.

1 A Spherical Triangle is a figure consisting of three arches of the greatest circles upon the superficies of a Sphere or Globe, every one being less than a Semicircle.

2 A great circle is that which divideth the Sphere, or Globe into two equal parts, and thus the Horizon, Equator, Zodiack, and Meridians before described, are

are all of them great circles: and of these circles or any other, there must be three arches to make a Triangle, and every one of these arches severally must
Fig. 1. be less than a Semicircle: for if in the Triangle A B C (compo ed of the arches A B the height of the Pole above the Horizon, A C an arch of the great Meridian, and B C an arch of the Horizon) you produce the sides A B and B C, till they meet in the point H, the arches B A H and B C H are each of them a Semicircle; the one, to wit, B A H being the half of the Brazen Meridian, and the other, *viz.* B C H being the half of the wooden Horizon, and therefore the arch B A or B C, is less than a Semicircle. In like manner, if the sides A B and A C be produced, the side A C may be also proved to be less than a Semicircle.

3 The Spherical or Circular lines, are either parallel or angular.

Fig. 3. 4 Parallel arches or circles, are such as are drawn upon the same center within, without, or equal to another arch or circle; thus the circles B F C and D C E are (though lesser circles) parallel to the Horizon, and the Horizon is a circle parallel to these Almicanters or circles of altitude; and these circles being drawn between the Equinoctial and the Poles are parallels of latitude, and the Equinoctial it self in that position is a circle parallel to the Horizon, or rather they doe both make but one circle.

5 A Spherical angle, is that which is conteined by two arches of the greatest circles upon the superficies of the Globe intersecting one another: angles made by the intersection of two little circles, or of a little circle with a great, we take no notice of in the Doctrine of Spherical Triangles.

6 A Spherical angle is either Right or Oblique.

7 A Spherical Right angle, is that which is conteined, by two arches of the greatest circles in the superficies of the Sphere cutting one another at right angles

angles, that is, the one being Right or perpendicular to the other; thus the brass Meridian cutteth the Horizon at Right angles, and thus the Meridians drawn upon the Globe as well as the Brass Meridian doe all of them cut the Equator at Right angles, and thus also the Quadrant of altitude being fixed in the Zenith, doth cut the Horizon at Right angles.

8 An Oblique Spherical angle, is that which is contained by two arches of the greatest circles in the superficies of the Sphere, not being right or perpendicular to one another.

9 An Oblique Spherical angle is either Obtuse or Acute.

10 An Obtuse Spherical angle, is that which is greater than a Right angle.

11 An Acute, is that which is lesser than a right.

12 If two of the greatest circles of the Sphere shall pass through each others Poles, those two great circles shall cut one another at right angles: thus the brazen Meridian doth intersect the Equinoctial and Horizon.

13 If two of the greatest circles of the Sphere shall intersect one another, and pass through each others Poles, they shall intersect one another at unequal or oblique angles, the angle upon the one side of the intersection being obtuse, or more than a right and the angle upon the other side of the intersection being acute or less than a Right. Thus the arch A C doth intersect the brass Meridian and the Horizon, but not in the Poles of either, therefore the angle H A C upon one side of the intersection of that arch with the Meridian is more than a Right angle, and the angle C A B upon the other side of the intersection is less: and so likewise the angle A C H, upon the one side of the intersection of the arch A C with the Horizon H B is greater than a Right angle, and the angle A C B upon the other side of the intersection is less. *Fig. 1.*

14 A Spherical angle is measured by the arch of a great circle described from the angular point between the sides of the angle, those sides being continued
Fig. 1. unto Quadrants; thus the arch of the Horizon H P is the measure of the angle H Z P at the Zenith, the sides H Z and P Z being Quadrants.

15 The complement of a Spherical arch or angle is so much as it wanteth of a Quadrant, if the arch or angle given be less than a Quadrant, or so much as it wanteth of a Semicircle, if it be more than a Quadrant.

16 An arch of a great circle cutting the arch of another great circle, shall intersect one another at Right angles, or make two angles, which being taken together, shall be equal unto two Right: thus the Quadrant of altitude being fixt in the Zenith as in the first Scheme doth cut the Horizon at Right angles, but the arch A C in the Triangle A B C of the same Scheme, doth cut the Horizon at oblique angles, making the one more, the other less than a Right angle, and both together equal to a Semicircle.

Now from these general Definitions proper to Spherical lines or arches, the general affections of these arches may easily be discerned. I mean the various positions of the Globe of the Earth in respect of all the singular Inhabitants thereof, which is threefold: I say that the whole body of the Sphere or Globe, in respect of the Horizon, is looketh upon by the Inhabitants of the Earth, under a triple constitution, *viz.* either of a *Parallel*, a *Right*, or an *Oblique* Sphere.

Fig. 3. A *Parallel* Sphere is, when one Pole of the World is elevated above the Horizon to the *Zenith*, the other depressed as low as the *Nadir*, and the Equinoctial line joyned with the Horizon, and they which there inhabite (if any such be) see not the Sun or other Star rising or setting, or higher or lower in their diurnal revolution; as may be perceived, by turning the

Glob

Globe with the brazen Meridian, ſo that one of the Poles may be in the Zenith, diſtant from the Horizon 90 deg. the other in the Nadir, and the Equinoctial line in the Horizon.

And ſeeing that the Sun traverſeth the whole Zodiack in a year; and that half the Zodiack is above the Horizon and half under it, it cometh to paſs, that the Sun ſetteth not with them, for the ſpace of ſix moneths, nor giveth them any light for the ſpace of other ſix moneths, and ſo maketh but one day and night, of the whole year.

A Right Sphere is, when both the Poles of the World do lie in the Horizon, and the Equinoctial circle is at his greateſt diſtance from it, paſſing through the Zenith, of the place. And in this poſition of the Sphere, all the Cœleſtial Bodies, Sun, Moon, Planets, and fixed Stars, by the daily turning about of the Heaven, do directly aſcend above, and alſo directly deſcend below the Horizon, becauſe the circles which they may in their firſt or daily motion do cut the Horizon perpendicularly and as it were at Right angles. *Fig.* 4.

In this poſition alſo, all the Stars may be obſerved to riſe and ſet in an equal ſpace of time, and to continue as long above the Horizon, as they do under it, the day and night to thoſe Inhabitants, being alwayes of an equal length.

By the Oblique Sphere is underſtood ſuch a conſtitution of the Heaven in which the Axis of the world (being neither direct nor parallel to the Horizon) is inclined obliquely towards both ſides of the Horizon, as in the ſecond Figure, by which you may ſee that this poſition of Sphere is ſhewed by the Globe, when the Axis doth not lie in the Horizon, not yet directly elevated, but declining obliquely *North* or *South* towards the Horizon, whence it cometh to paſs, that ſo much as one of the Poles is elevated above the Horizon, upon the one ſide, ſo much *Fig.* 2.

P is

is the other depressed, under the the Horizon upon the other side.

And in this position of the Sphere, the dayes are sometimes longer than the nights, sometimes shorter and sometimes of equal length: when the Sun is in either of the Equinoctial points, the dayes and nights are equal, but when he declineth from the Equator, towards the elevated Pole, the dayes are observed to increase; and when he declineth from the Equator towards the opposite Pole or the Pole depressed, the dayes do decrease, as is manifest as well by the Figure as the Globe it self, for when the Sun riseth at C, the line M C is the semidiurnal arch of the longest day, when he riseth at D, the dayes and nights are of equal length, and when he riseth at P, the dayes are shortest in those places of the world in which the Pole is thus much elevated above the Horizon as the arch A B in the Scheme.

CHAP. XII.

Of the kinds and parts of Spherical Triangles with the solution of them.

HAving shewed what a Spherical Triangle is, and of what circles it is composed, with the general affections of such lines: we will now shew how many several sorts of Triangles there are, of what circular parts they do consist, and such affections proper to them as will render the solution of them more clear and certain.

1 Spherical Triangles are either Right or Oblique.

2 A Right angled Spherical Triangle is that which hath one or more Right angles.

Fig. 1. 3 A Spherical Triangle which hath three Right angles, hath alwayes his three sides Quadrants. As in

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Fig: 5

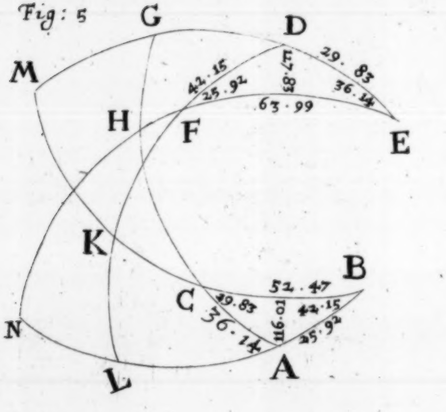
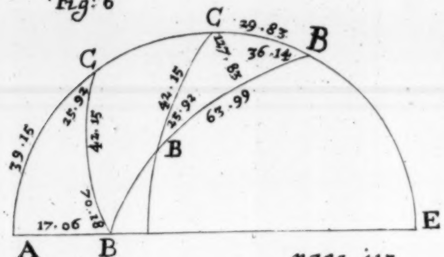


Fig: 6



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in the Triangle ZDH, the angles HZD, ZDH and ZHD are Right angles, and the three sides ZH, ZD and HD are Quadrants.

4 A Triangle that hath two Right angles, hath the sides opposite to those angles Quadrants, and the third side is the measure of the third angle. As the side ZH and ZP, in the Triangle ZHP are Quadrants, and cut the Horizon at Right angles in the points H and P. and the arch HP is the measure of the angle HZP, so that of those Triangles which have more than one Right angle, there seldom ariseth any Question. But the Right angled Triangle, which hath one Right and two acute angles, is that which cometh most commonly to be resolved. *Fig.* 1.

5 The legs of a Right angled Spherical Triangle are of the same affection with their opposite angles. *Fig.* 1.

Thus he side ZB in the Triangle ZDB is a Quadrant, and therefore the angle at D is a Quadrant also, because Z is the pole of the arch DB, and ZD perpendicular thereunto; and in the Triangle BDÆ the side BÆ being more than a Quadrant the angle BDÆ is more than a Quadrant also, it being more than the Right angle, ZDB, and in the Right angled Spherical Triangle ADB the side AB being less than a Quadrant; the angle ADB is also less than a Qnadrant, it being less than the Right angle ZDB.

6 An Oblique angled Spherical Triangle, is either acute or obtuse.

7 An acute angled Spherical Triangle hath all its angles acute.

8 An Obtuse angled Spherical Triangle hath all its angles either obtuse or mixt, *viz.* acute and obtuse.

9 In any Spherical Triangle whose angles are all acute, each side is less than a Quadrant.

10 The sides of a Spherical Triangle may be turned into angles, and the angles into sides, the complements

ments of the greateſt ſide or greateſt angle to a ſemicircle, being taken in each converſion.

Fig. 5. Let A B C be a Spherical Triangle, obtuſe angled at A, let K L be the meaſure of the angle at B, & M G the meaſure of the angle at C, and N H the meaſure of the angle H A N which is the complement of the obtuſe angle C A B being the greateſt angle in the given Triangle, K L is equal to the arch F D, becauſe L F and K D are Quadrants, and their common complement is K F.

Secondly, M G is equal to D E, becauſe M D and G E are Quadrants, and G D their common complement.

Thirdly, N H is equal to D E, becauſe N D and H E are Quadrants and H D their common complement, the ſides therefore of the Triangle F D E are equal to the angles of the triangle A B C.

Again, G H is the meaſure of the angle at E and equal to C A & M K is the meaſure of M D K & equal to C B, and L N is the meaſure of F and equal to A B, and therefore the angles of the triangle E D F are equal to the ſides of the triangle A B C as was to be proved.

11 In Right angled ſpherical triangles the ſides including the Right angle we call the legs, the ſide ſubtending it, the Hypotenuſa.

12 In oblique angled triangles, the ſides comprehending the angle given or ſought; we call the legs as before, and the third ſide we call the baſe.

13 In every ſpherical triangle (beſides the Area or ſpace conteined) there are ſix conteining parts, *viz.* three ſides, and three angles, and of theſe ſix there muſt be alwayes three given, to find the reſt.

14 In Right angled ſpherical triangles, there are but five of the ſix parts, which come into queſtion; the three ſides and two acute angles: becauſe the third being right, is alwayes known, of theſe five parts any two being given the reſt may be found.

15 In

15 In Right angled ſpherical triangles, there are 16 Caſes or Problems, ſix for finding the legs, four for the hypotenuſa, and ſix for the angles: but here we may reduce them to five, becauſe the Globe doth alwayes reſolve two of them at once; for by two parts given you may preſently find two of three parts unknown, and by turning the triangle the third alſo.

16 A right angled ſpherical triangle, may be repreſented upon the Globe in this manner: Elevate one of the Poles of the Globe above the Horizon to the quantity of one of the given legs, ſo ſhall the diſtance between the Equinoctial and the Zenith be equal thereunto, and at the Zenith faſten the Quadrant of altitude, and number from the braſs Meridian upon the Equinoctial the quantity of the other leg, to the extremity whereof move the Quadrant of altitude, ſo ſhall there be delineated upon the Globe the Right angled ſpherical triangle Æ Z R, as may be ſeen in figure 1.

PROBLEM 1.

The legs of a Right angled ſpherical triangle being given, to find the reſt.

IN the Right angled ſpherical triangle A B C, let there be given the leg A B *Fig. 6.*

And the leg A C

And let there be required.

The Hypotenuſa B C

The angle { A C B, A B C

Number one of the given legs A B, from the interſection of the Equinoctial with the Meridian at Æ, to the Zenith at Z, and the other leg A C number upon the Equinoctial from Æ to R, then faſten the Quadrant of altitude in the Zenith at Z and move it to R, then ſhall the arch R Z give the hypotenuſa *Fig. 1.*

BC, and the arch HP in the Horizon shall be the measure of the angle ÆZR, or the angle ABC.

And to find the angle ACB, number the leg AC from the intersection of the Equinoctial with the Meridian at Æ to the Zenith at Z, and the other leg AB, upon the Equinoctial from Æ to R, then fix the Quadrant of altitude in the Zenith at Z, and move it to R, so shall the arch HP in the Horizon be the measure of the angle in the Zenith ÆZR, or the angle ACB inquired.

In this Probleme then three Questions may be propounded and resolved.

The legs AB and AC being given, we may find, 1 The hypotenusa BC, 2 Angle, ABC, 3 Ang. ACB.

PROBLEM 2.

Fig. 6. *The Hypotenusa and one leg given, to find the rest.*

IN the Right angled spherical triangle ABC, let there be given

The Hypotenusa BC	To find	Leg	AB	
The leg AC		Angle	ACB	ABC

Fig. 1. Number the given leg AC upon the brasse Meridian from the intersection of the Equinoctial at Æ to the Zenith at Z, and there fasten the Quadrant of altitude, upon which from the Zenith downward, number the hypotenusa ZR, and move the Quadrant of altitude ZP until the point R doth intersect the Equinoctial ÆD, so shall ÆR the arch of the Epuinoctial, be the quantity of the leg AB, and HP in the Horizon shall be the measure of the angle ÆZR equal to the angle ABC.

And to find the angle ABC, number the given leg AG in the Equinoctial from Æ to R, and the hypotenusa

nusa B C upon the Quadrant of altitude as before, and apply the same to the point R, in the Equinoctial, and where the point Z shall touch the Meridian fasten the Quadrant of altitude, then if you elevate the pole A until A B shall be equal to Æ Z, the point Z shall be in the Zenith, and the arch H P in the Horizon shall be the measure of the angle Æ Z R equal to the angle A B C.

In this Problem 6 Questions may be propounded and resolved. For

The Hypotenusa B C and the leg A C being given, may find,

1 The leg A B, 2 Angle A C B, 3 Angle A B C.

Or, The hypotenusa B C and the leg A B being given, we may find,

1 The leg A C, 2 Angle A C B, 3 Angle A B C.

PROBEM 3.

The hypotenusa and angle given, to find the rest. *Fig. 6.*

IN the Right angled spherical Triangle A B C, let there be given

The hypotenusa B C	To find	The angle A B C		
The angle A C B		The legs	A B	
			A C	

Number the given angle A C B in the Horizon from H unto P, and unto P apply the Quadrant of altitude, *Fig. 1.* in which having also numbred the hypotenusa B C from Z to R, move the Pole A higher or lower, untill the Equator Æ D, doth intersect the Quadrant of altitude in the point R, and then fasten the Quadrant in the Zenith, so shall the arch Æ Z give you the quantity of A C, and Æ R the quantity of A B.

And to find the angle A B C, you must make a mark upon the Globe at the point R, and another at Z, then bring the point R to the Zenith, and there fasten the Quadrant of altitude, which being moved to the point Z

Z will cut the Horizon at the quantity of the angle A B C.

In this Problem 6 Questions also may be propounded and resolved. For,

The hypotenusa B C and the angle A C B being given, we may find,

1 The leg A B, 2 the leg A C, 3 the ang. A B C.

Or, the hypotenusa B C and the ang. A C B being given,

We may find { 1 The leg A B
2 The leg A C
3 The ang. A C B.

PROBLEM 4.

A leg and an angle given, to find the rest.

Fig. 6.

IN the right angled triangle A B C, let there be given, the leg A B, and the angle thereunto adjacent A B C

To find the { other { leg A C
angle A C B
Hypotenusa B C

Fig. 1. Number the given leg A B from the intersection of the Equinoctiall with the Meridian at Æ to Z, and the given ang. A B C in the Horizon from H to P, and then fasten the Quadrant of altitude in the Zenith at Z, and move it to the point P in the Horizon, then shall Æ R be equal to the leg A C, and Z R shall be equal to the hypotenusa B C.

And to find the ang. A C B make a mark upon the Globe at Z, and bring the point R to the Zenith, & there fasten the quadrant of altitude which being moved to the point made upon the Globe, will give in the Horizon the measure of the ang. A C B.

But if the leg given be A B, and the ang. A C B opposite thereunto, to find, 1 The hypotenusa B C.

The other { leg A C
angle A B C

Number

Number the given leg A B upon the Equinoctial, from Æ to R, and the ang. A C B in the Horizon from H unto P, then turn the Globe till the Quadrant of altitude touching P and R, will also reach the Zenith, and there fasten it, then will Z R be the hypotenusa B C & Æ Z the leg A C, & the ang. A B C may be found as the ang. A C, B in the last Example.

In this Problem 12 Questions may be propounded and resolved, For,

A B & A B C	may be given,	B C.	2 A C.	3 A C B	
A B & A C B	to find	B C.	2 A C.	3 A B C	
A C & A C B	may be given,	B C.	2 A B.	3 A B C	
A C & A B C	to find	B C.	2 A B.	3 A C B	

PROBLEM 5.

The oblique angles being given, to find the rest.

IN the right angled spherical triangle A B C, let the oblique angles A C B and A B C be given, *Fig.* 6.

To find the Hypotenusa B C; Legs A B, A C

For the resolving of this Problem, we must look back to the tenth of this Chapter, were you were taught to convert the sides of any Spherical triangle, into the angles of another, and the angles of that Triangle into the sides of the former, and where you may see, that the side F D is equal to the oblique ang. A B C: the side D E is equal to the oblique ang. A C B, and the side F E is equal to the complement of the angle B A C. *Fig.* 5.

Now then if you number the side D E upon the brass Meridian from the Pole A to the Zenith at Z, and the side F E 90, upon the great Meridian, upon the Globe from A unto R, and the side F D upon the Quadrant of altitude, being fastened at the Zenith from Z *Fig.* 1.

 down-

downwards towards P, and move the Globle and the Quadrant of altitude until the great Meridian doth intersect it, so as that that arch comprehended between the great Meridian and the Zenith be equal to the side F D, then shall Æ R be the measure of the ang. R A Z equal to the side A C, and the arch H P in the Horizon, shall be the measure of the angle Æ Z R equal to the hypotenusa B C, and the angle A R Z being measured, (as hath been shewed already) shall be equal to the side A B.

And in this Problem 3 Questions may be propounded and resolved: For the oblique angles A C B and A B C being given, We may find 1 The Hypotenusa B C, 2 The leg A B, 3 The leg A C.

And therefore in every Right angled Spherical Triangle there are 30 Questions, the which (as hath been shewed) may be resolved by these 5 Problems, and are expressed in the following Table.

	Data		*Quaesita*		
Q	*Legs*	*Ang.*	*Legs*	*Ang.*	*Resolved*
3	*A B: A C*		*B C*	*B . C*	by Probl. 1.
6	*A B* *B C* *A C*		*A C* *A B*	*B . C*	by Probl. 2
6	*B C*	*A B C* *A C B*	*A B: A C*	*A C B* *A B C*	by Probl. 3.
12	*A B*	*A B C* *A C B*	*B C: A C*	*A C B* *A B C*	by Probl. 1.
	A C	*A C B* *A B C*	*B C: A B*	*A B C* *A C B*	
3		*B : C*	*A B* *B C* *A C*		by Probl. 5.

CHAP. XIII.

Of the Solution of the Oblique angled Spherical Triangles.

THough the Problems following, are applied to Oblique angled Spherical triangles onely, yet are the Rules general, and may very well serve for the solution of all Spherical triangles whatsoever.

In oblique angled Spherical triangles there are 12 Cases, but in resolving them by the Globe they are reduced unto 6 Problmes.

An oblique angled spherical triangle may be represented upon the Globe, in this manner: Number one of the given sides from one of the Poles elevated above the Horizon to the Zenith: and there fasten the Quadrant of altitude, upon which from the Zenith downwards, towards the Horizon, number another side, and the third side upon the great Meridian, from the Pole towards the Equinoctial, then turn the Globe till the side numbred upon the Quadrant of altitude, and the side numbred upon the great Meridian shall intersect each other, so shall there be delineated upon the Globe the oblique angled spherical triangle A Z E, as may be seen in the 1 Scheme.

PROBLEM 1.

The three sides given, to find the angles.

IN an oblique angled spherical triangle BCD, let the given sides be CB. CD. and BD. to find the angles C. D. and B. *Fig.* 6.

Fig. 1. Number the side C D upon the brass Meridian, from A to the Zenith at Z, the side B D upon the great Meridian from A to E, and the side C B upon the Quadrant of altitude from Z to E, then shall the arch of the Horizon P B be the measure of the ang. A Z E equal to B C D.

And the arch in the Equator Æ S, shall be the measure of the ang. E A Z equal to B D C.

Lastly, to find the ang. C B D you may number the side B C from A to Z, the side C D from Z to E, and the side B D as before, then shall the arch of the Equator Æ S be the measure of the angle C B D inquired.

Or thus, make a mark upon the Globe at Z, and bring the point E to the Zenith, then shall the quadrant of altitude (being fastened in the Zenith, and laid to the said point upon the Globe) give in the Horizon from H unto P, the measure of the angle A E Z, as before.

In this Problem 3 Questions may be propounded and resolved, for the three sides B C. C D. and D B, being given, we may find the angles B C D. C D B. and D B C.

PROBLEM 2.

Two sides and their conteined angle being given, to find the rest.

Fig. 6. IN the oblique angled spherical triangle B C D, the given sides are B C. C D. angle B C D.

To find the { Third side B D; other angles { C D B, D B C

Fig. 1. Number the side C D, from A unto Z, upon the brass Meridian, bring the point Z to the Zenith, and there fasten the Quadrant of altitude, in which from Z downwards towards the Horizon, number the side C B

CB : thirdly, number the given angle B C D in the Horizon from B unto P, and thither bring the Quadrant of altitude, lastly, bring the great Meridian upon the Globe to the point E in the Quadrant of altitude, so shall the arch A E be equal to the side B D, and the arch in the Equator Æ S, shall be the measure of the ang. E A Z equal to the ang. C D B.

In this Problem 9 Questions may be propounded and resolved.

Let there be given { B C. C. C D / C D. D. D B / D B. B. B C } to find { B. B D. D / B. B C. C / C. C D. D }

PROBLEM 3.

Two angles and a side between them being given, to find the rest.

In the Oblique angled spherical triangle B C D. *Fig. 6.*

Let there be given { angles { B C D / C D B } / side C D }

To find the ang. B, and the sides { B C / B D }

Number the side C D from *A* unto Z, then bring the point Z into the Zenith, and there fasten the Quadrant of altitude: number also the ang. C D B, in the Equinoctial from Æ unto S, to which point bring the great Meridian upon the Globe: lastly number BCD upon the Horizon from B unto P, and bring thither the Quadrant of altitude, then shall *A* E be equal to the side B D, and Æ Z shall be equal to the side BC. And the angle *A* E Z equal to the angle C B D may be found, as hath been already shewed in the first Problem. *Fig. 1.*

In this Problem 9 questions also may be propounded and resolved, for we may have given,

B. BC. C	to find	CD. D. D*B*
C. CD. D		D *B*. *B*. *B*C
D. D B. B		*B* C. C. CD

PROBLEM 4.

Two sides and an angle opposite to one of them being given, to find the rest.

Fig. 6. In the Oblique angled spherical triangle *B*CD.

The given	sides	C D	The inquired	side BD		
		C *B*		angles	C	
	angle C B D				D	

Fig. 1. Number the given side CB upon the brass Meridian from *A* unto Z, bring the point Z into the Zenith, and there fasten the Quadrant of altitude, upon which number the other side CD, from Z to E, and the ang CBD opposite thereunto in the Equinoctial from Æ unto S, then bring the great Meridian upon the Globe to the point S also, so shall the arch *A* E be equal to the inquired side DB, and the arch BP in the Horizon the measure of the angle DCB, the ang. at E equal to CBD may be found, as hath been shewed.

In this Problem 18 questions may be propounded and resolved. For we may have given,

CD. DB. B		*BC. C. D*
CD. DB. C		*BC. B. D*
DB. BC. C	To find	*CD. D. B*
DB. BC. D		*CD. C. B*
BC. CD. D		*DB. B. C*
BC. CD. B		*DB. D. C*

PRO-

PROBLEM 5.

Two angles and a side opposite to one of them being given, to find the rest.

In the Oblique angled spherical triangle B C D. *Fig.* 6.

The given { angles { C, D }, side CB 30 } The inquired { sides { CD, D B }, angle CBD }

Number the given ang. CDB, which is opposite to the side given upon the Equinoctial from Æ to S, and thither bring the great Meridian upon the Globe, and the other given angle BCD, number in the Horizon from *B* unto P, and the given side C*B*, number upon the Quadrant of altitude from Z to E, and then set the beginning of the Quadrant upon P in the Horizon, and let the mark upon it at E, cross the great Meridian A S: Lastly, move the Pole of the world higher or lower, till the screw end of the Quadrant fall exactly upon the Zenith at Z, which being fastened there, the arch of the brass Meridian *A* Z shall be equal to the side CD and the arch of the great Meridian *A* E to the side BD, and the angle C B D may be measured, as was shewed before. *Fig.* 1.

In this Problem 18 questions may be propounded and resolved: for we may have given,

C. D. D B } *C. D. B C* }		{ *C D. C B. B* { *C D. D B. B*
D. B. B C } *D. B. C D* }	To find	{ *C D. D B. C* { *B C. D B. C*
B. C. C D } *B. C. B D* }		{ *B C. B D. D* { *B C. C D. D*

PROBLEM 6

The three angles being given, to find the rest.

In the Oblique angled spherical triangle B C D.

The

The given angles are BCD. CDB and DBC.

The sides inquired are CD. DB and BC.

To resolve this Problem, as in the 5 *Probl.* of right angled spherical triangles, so here we must convert the angles into sides, and then we find the angles, as was shewed in the 1 *Probl.* those angles shall be the sides of the triangle BCD inquired.

In this Problem then three Questions may be propounded and resolved: and therefore in every oblique angled spherical triangle there are 60 Questions, as may be seen in the Table.

BC. CD. DB	*by Probl.* 1 3	*B.* C. *D*
BC. C. CD *CD. D. DB* *DB. B. BC*	*by Probl.* 2 9	*B. BD. D* *B. BC. C* *C. CD. D*
B. B C. C *C. C D. D* *D. D B. B*	*by Probl.* 3. 9	*CD. D. D*B *D*B. *B.* B*C* B*C. C. CD.*
*CD. D*B. B *CD. D*B.*C* *DB. BC. C* *DB.* B*C. D* *BC. CD. D* B*C. CD. B*	*by Probl.* 4 18	B*C. C. D* *BC.* B. *D* *CD. D.* B *CD. C.* B *DB.* B. *C* *D*B. *D. C*
C. D. DB *C. D.* B*C* *D. B. BC* *D.* B. *CD* B.*C. CD* *B.C.* B *D*	*by Probl.* 5. 18	*CD. C*B. *B* *CD. D* B.*B* *CD. D*B.*C* B *C. D* B.*C* B*C. DB. D* B*C. CD. D*
B. C. D	*by Probl.* 6. 3	B*C*. CD. DB

CHAP

CHAP. XIV.

Of the Spherical Problems both Astronomical *and* Geographical.

WE are now arrived at that, which at the first was chiefly intended, the several & distinct Questions, in which the use of the Globe may best appear: and those are either such as concern the Heavens or the Earth and Seas; the things resolveable upon the Globe, which do concern the Heavens, we call *Astronomical Problems*, and those which concern the Earth and Seas *Geographical*.

The Problems *Astronomical* are very numerous, the most useful and necessary, are these which follow.

1 To find the longitude of the sun, with the longitude and latitude of any star.

2 The declination of the sun and any star.

3 The right ascension of the sun or any star.

4 The ascensional difference, with the oblique ascensions and descensions of the sun or stars.

5 The azimuth or distance of the sun or stars, from the *N.* or *S.* part of the Horizon.

6 The amplitude or distance of the sun or stars, from the *E* or *W* points of the Horizon.

7 The semidiurnal and seminocturnal arches of the sun or stars, the time of their rising and setting, and the length of their day and night.

8 The hour of the day by the sun, or of the night by the stars.

9 The beginning and ending of twilight.

10 The altitude of the sun or star.

11 The angle of the ecliptick with the Equinoctial.

12 The angle of the suns meridian with the Ecliptick.

13 The angle of the Ecliptick with the Horizon.
14 The angle of position of the sun or star.

The Problems Geographical are chiefly four.

1 The latitude of any place, or the height of either Pole above the Horizon.

2 The longitude or distance of any place from the great Meridian.

3 The distance between any two places.

4 The way, otherwise called the Course or Rumb leading from the one to the other.

How and how many several wayes these Problems may be resolved upon the Globe, will best appear, by considering the several triangles, in which these *Problems* are conteined if not expressely, yet at least vertually.

And in the Sphere or Globe there are eight triangles, by the knowledge whereof most Problems (whether *Astronomical* or *Geographical*) may be resolved, and these which we have here set down may be resolved by them several wayes.

Of these eight Triangles five are rightangled, which here shall have their denomination from their Hypotenusaes.

1 The Ecliptical triangle, whose hypotenusa is an arch of the Ecliptick, his legs are arches of the Equator, and Meridian, this is represented upon the Globe, by the triangle D E F, in which the five circular parts be sides the right angle are.

Fig. 7. 1 The hypotenusa, or arch of the Ecliptick D E.

2 The leg or arch of the Equator D F.

3 The leg or arch of the Meridian F E.

4 The Oblique angle of the Equator with the Ecliptick or the suns greatest declination E D F.

5 The oblique angle of the Ecliptick and Meridian, or angle of the suns position D E F.

Of

Of these circular parts, the angle of the suns greatest declination E D F is still the same, and in all triangles wherein it is found, it is one of the terms supposed to be given, and yet to make up the number of Problems in those triangles, we shall here sometimes propound it, as a term that is inquired: but before we proceed to the particular enumeration of those Problems, we will first shew what is meant by the declination of the sun or other star: and then the usual way by which the suns greatest declination is first found.

The declination of the sun or other star, is his or their distance from the Equator, that is, from that circle which being drawn between the Poles divideth the Globe into two equal parts, and as they decline from thence either Northward or Southward, so is their declination nominated *North* or *South*; and hence it is apparent, that A Z the complement of A B, the Poles elevation, is equal to Æ H the height of the Equator; and H M the suns greatest Meridian altitude, is equal to the sum of Æ H the height of the Equator, and Æ M the Suns greatest declination; from the Equator towards the *North*. And Æ T the suns greatest declination from the Equator towards the *South* Pole is the difference between, Æ H the height of the Equator, and H T the suns least Meridian altitude: and hence it is apparent that the suns greatest declination is the half of T M the difference between M H the suns greatest, and H T the suns least Meridian altitude, from whence the height of the Equator and consequently the height of the Pole above the Horizon may thus be found.

			Deg.
	At London		
The Suns { *greatest*	{ *Meridian*	} H M.	61.99167
least }	{ *altitude is*	} H T.	14.94167
Their difference is		M T.	47.05000
Half that difference is equal to T Æ *or* M Æ *the suns greatest declination*		}	23.52500

Which

Which being deducted from H M the ſuns greateſt meridian altitude, the remainer is H Æ 38.4667 the height of the Equator, and the complement of H Æ 38.4667 to 90, is Z Æ = A B 51,5333 the height of the Pole above the Horizon.

Next to the ſuns greateſt declination, his diſtance from the next Equinoctial point or place in the Ecliptick, called his longitude is uſually given, and as to our purpoſe is readily enough found, right againſt the day of the moneth in the Kalender upon the Horizon, which being found there, muſt be alſo found in the Zodiack, and in this triangle is repreſented by the arch D E. And theſe two parts of that triangle being given, the legs F D and F E repreſenting the ſuns right aſcenſion, and preſent declination, may be found by bringing the point E to the braſs Meridian for then the arch of the Equinoctial from D to the braſs meridian ſhall be the ſuns right aſcenſion, and the arch of the braſs meridian interſected between the Equinoctial circle, and the Ecliptick ſhall be the ſuns preſent declination, ſo that in this triangle there is nothing more to be inquired, but the oblique angle of the Ecliptick with the meridian, or the angle of the ſuns poſition D E F, and this may be found by bringing the point E to the braſs meridian, and turning the Globe till it fall alſo in the Zenith, for then the arch of the Horizon intercepted between the braſs meridian, and the Ecliptick, ſhall be the meaſure of the angle of the ſuns poſition D E F inquired.

And thus having ſhewed how the parts of this triangle may be ſeverally found, we will now ſet down the 30 Problems conteined in this triangle, and by which of the Problems of right angled ſpherical triangles they may be reſolved, and leave it to the Readers diſcretion, from the ſeveral Datas, to find the thing inquired, either in its own proper place, or by thoſe Problems by which all right angled ſpherical triangles are to be reſolved in general.

The

The Right ascension F D, and the present declination of the Sun F E being given, there may be found,

1 The suns longitnde D E
2 The suns greatest declination F D E } *by 1 Probl.*
3 The angle of the suns position D E F

The suns longitude D E, and right ascension D F being given, there may be found,

4 The suns present declination F E
5 The suns greatest declination F D E } *by 2 Probl.*
6 The angle of the suns position F E D

The suns longitude D E, and his present declination F E being given, there may be found,

7 The suns right ascension D F
8 The ang. of the suns greatest declination F D E
9 The angle of the suns position F E D
by the 2 Probl.

The suns longitude D E and greatest declination F D E being given, there may be found,

10 The suus right ascension D F
11 The suns present declination F E } *by 3 Probl.*
12 The angle of the suns position F E D

The suns longitude D E, and his angle of position F E D being given, there way be found,

13 The suns right ascension D F
14 The suns present declination F E
15 The angle of the suns greatest declination F D E.
by the 3 Problem.

The suns right ascension D F, and angle of his greatest declination F D E being given, there may be found,

16 The suns longitude D E
17 The suns present declination F E } *by the 4 Probl.*
18 The suns angle of position D E F

The suns right ascension D F and angle of the suns position D E F being given there may be found,

19 The suns longitude D E
20 The suns present declination F E } *by 4 Prob.*
21 The ang. of the ☉ greatest de cli. F D E

 The

The suns present declination F E, and angle of position D E F being given, there may be found,

22 The suns longitude D E
23 The suns right ascension D F
24 The ang. of the suns greatest declination F D E. *by the 4 Problem.*

The suns present declination F E, and the angle of the suns greatest declination F D E being given, there may be found

25 The suns longitude D E
26 The suns right ascension D F } *by 4 Probl.*
27 The suns angle of position D E F

The angles of the suns greatest declination F D E, and of position D E F being given, there may be found,

28 The suns longitude D E
29 The suns right ascension D F } *by 5 Probl.*
30 The suns present declination F E

CHAP. XV.

The suns Meridian altitude, and present declination given, to find the Poles elevation,

THis is one of the most useful Problems in Navigation, not actually conteined in the Ecliptical Triangle, and yet may conveniently enough be referred to it, because the present declination of the sun is several wayes found by it, which being added too, or subtracted from, the suns Meridional altitude according to the following directions the sum or difference is the height of the Equator, whose complement unto 90 is the height of the Pole.

In this Problem there are seven varieties.

The first, is when the sun hath no declination; as when he is in the Equator, that is, in the beginning of *Aries* and *Libra*, and then the complement of his Meridian altitude unto 90, is the height of the *North* Pole, if the sun be on the *South* side of the Meridian, or of the *South* Pole, if on the *North*.

The second variety is when the suns Meridian altitude is just 90, and then the declination it self, is the elevation of the *North* Pole, if that be *North*, and of the *South* Pole if the declination be Southward.

The third variety is when the suns Meridian altitude is less than 90 upon the *South* side of the meridian, and have *South* declination. In this Case, Adde the suns meridian altitude to his present declination, their sum is the height of the Equator, and that complement to 90 the height of the *North* Pole.

The 7th of *February* 1660 the *South* declination of the Sun is	11.96
The suns meridian altitude suppose	15.27
There sum or total	27.25
The height of the *North* Pole	62.75

But if the sum of the suns declination and meridian altitude exceed 90, subtract 90 from it, the remainer is the altitude of the *South* pole.

As admit the suns *South* declin. to be	11.96
And his meridian altitude	87.23
There sum is	99.19
Therefore the height of the *South* pole is	9.19

The fourth variety is when the suns meridian altitude is less than 90 upon the *South* side of the meridian, and hath *North* declination; in which Case the suns declination being subtracted from his meridian altitude,

tude, there difference is the height of the Equator, and the complement unto 90, the elevation of the *North* pole.

The fifth variety is when the suns meridian altitude is less than 90, upon the *North* side of the meridian, and hath *North* declination; In this Case the suns declination being added to the suns meridian altitude, their sum or aggregate is the height of the Equator, and the complement thereof to 90 the height of the *South* pole.

But if their sum be more than 90, subtract 90 from it, the remainer is the altitude of the *North* pole.

The sixth variety is when the suns meridian altitude is less than 90 upon the *North* side of the meridian, and his declination *South*: In this Case, the sun declination being subtracted from his meridian altitude, the remainer is the height of the Equator, and the complement thereof to 90 is the height of the *South* pole.

The seventh variety is when the sun or stars is observed to be upon the merdian under the Pole within the *Artick* or *Antartick* circles, in which place, the sun or stars declination must be subtracted from 90, what remaineth is the suns distance from the Pole, which being added to the meridian altitude observed, the sum or total is the latitude.

CHA. XVI.

Of the two Meridional Right angled Spherical triangles upon the Globe.

THere are two other Right angled spherical triangles besides the ecliptical, which I call Meridional because the hypotenusaes in them both, are arches of a meridian.

One

One of these is noted with the letters A B C, in which the five circular parts are. *Fig.* 7.

1 The hypotenusa or arch of a Meridian *A* C.

2 The leg or arch of the Horizon C B, or *Azimuth* of the sun from the *North*.

3 The leg or arch of the brass meridian, representing the height of the Pole *A* B.

4 The oblique angle of the meridian upon the Globe with the brass meridian, or angle of the hour from midnight B *A* C.

5 The Oblique angle of the suns meridian with the Horizon *A* C B or complement of the Suns angle of position.

In every of the 30 Problems conteined in this triangle, the quantity of the sides is given by inspection upon the Globe, the angle of the hour from midnight B A C, may be found either by the arch of the Equator intercepted between the two Meridians, or by the arch of the Horizon, the Pole A being first brought into the Zenith, and the angle of the suns meridian with the Horizon A C B may be measured as hath been shewed in the 12th Chapter, whether we shall referre the Reader for the solution of the Pcoblems following: In this triangle then,

The Poles elevation A B, *and Suns Azimuth from the North* B C *being given, there may be found.*

1 The arch of a meridian or complement of the suns declination A C.

2 The hour from midnight B A C.

3 The angle of the suns meridian with the Horizon A C B. *by the* 1. *Probl.* 12 *Chap.*

The Suns Azimuth from the North B C, *and complement of the Suns declination A C being given, there may be found,*

4 The elevation of the Pole A B

5 The hour from midnight B A C

6 The angle of the suns Meridian with the Horizon A C B. *by the 2 Probl. 12 Chapt.*

The Poles elevation A B, *and complement of the Suns declination* A C *being given, there may be found,*

7 The suns Azimuth from the *North* B C.
8 The hour from midnight B A C.
9 The angle of the suns Meridian with the Horizon A C B, *by the 2 Probl. 12 Chapter.*

The complement of the suns declination A C, *and the hour from midnight* B A C *being given, there may be found,*

10 The suns azimuth from the *North* B C.
11 The Poles elevation A B.
12 The suns meridian with the Horizon A C B. *by the 3 Problem 12 Chapter.*

The complement of the Suns declination A C, *and angle of the Suns Meridian with the Horizon* A C B *being given, there may be found,*

13 The suns azimuth from the *North* B C.
14 The hour from midnight C A B
15 The Poles elevation A B. *by the 3 Probl. 12 Chap.*

The Suns Azimuth from the North C B, *and angle of the Suns Meridian with the Horizon* A C B *being given, we may find,*

16 The complement of the Suns declination A C.
17 The hour from midnight C A B.
18 The Poles elevation A B. *by the 4 Probl. 12 chapt.*

The Suns Azimuth from the North B C, *and the hour from midnight* C A B, *being given, we may find,*

19 The complement of the Suns declination A C.
20 The angle of the suns Merid. with the Hor. ACB
21 The Poles elevation A B. *by the 4 Probl. 13 chap*

The

The Poles elevation A B, *and the hour from midnight* B A C *being given, we may find,*

22 The complement of the suns declinatiou A C.
23 The suns azimuth from the *North* B C.
14 The angle of the suns meridian with the Horizon A C B. *by the 4 Probl. 12 chap.*

The Poles elevation A B, *and angle of the Suns Meridian with the Horizon A C* B *being given, we may find,*

25 The complement of the suns declination A C.
26 The suns azimuth from the *North* B C.
27 The hour from mid-night B A C. *by 4 Pr. 12 ch.*

The hour from midnight C A B, *and the angle of the Suns Meridian with the Horizon* A C B *being given, we may find,*

28 The complement of the Suns declination A C.
29 The Suns azimuth from the *North* B C.
30 The Poles elevation A B. *by the 5 Probl. 12 chap.*

The other Right angled Meridional Triangle is noted with the letters D G H, in which the 5 Circular parts, are

Fig. 7.

1 The Hypotenusa or present declination of the Sun D G.
2 The leg or amplitude of the Sun at the hour of six D H.
3 The Suns height at the same time G H.
4 The angle of the Poles elevation G D H.
5 The angle of the Suns position D G H.

And in every of the 30 Problems conteined in this triangle, the quantity of the sides is given by inspection, A B is the measure of the angle G D H. And the angle of the Suns position D G H may be measured as hath been shewed in the 12 Chapter, whether we refer the Reader for the solution of the Problems following.

In

In this triangle then

The Suns amplitude DH, and altitude GH being given, we may find,

1 The Suns present declination DG
2 The Poles elevation GDH.
3 The angle of the suns position DGH. *by the 1 Problem 12 chapt.*

The Suns amplitude DH, and the Suns declination DG being given, we may find,

4 The Suns height at the hour of six GH.
5 The Poles elevation GDH.
6 The angle of the Suns position DGH.
by the 2 Probl. 12 chapter.

The Suns altitude GH, and the Suns declination DG being given, we may find,

7 The Suns amplitude DH.
8 The Poles elevation GDH.
9 The angle of the Suns position DGH.
by the 2 Probl. 12 chap.

The Suns declination DG, and the Poles elevation GDH being given, we may find,

10 The Suns amplitude DH
11 The Suns altitude GH.
12 The Suns angle of position DGH.
by the 3 Probl. 12 chap.

The Suns declination DG, and the angle of position DGH being given, we may find,

13 The Suns amplitude DH.
14 The Suns altitude GH.
15 The Poles elevation GDH. *by 3 Probl. 12 chap.*

The

The Suns amplitude D *H*, *and the Poles elevation* G D *H* *being given, we may find,*

16 The Suns declination D G.
17 The Snns altitude G H.
18 The Suns angle of position G D H. *by the 4 Probl. 12 chapter.*

The Suns amplitude D *H*, *and angle of the Suns position* D G *H being given, we may find,*

19 The Suns declination D G.
20 The Suns altitude G H.
21 The Poles elevation G D H. *by 4 Probl. 12 chap.*

The Suns altitude G H, and angle of the Suns position D G *H* *being given, we may find,*

22 The Suns declination D G
23 The Suns amplitude D H.
24 The Poles elevation G D *H*: 4 *Prob.* 12 *chap.*

The Suns altitude G H, and the poles elevation G D H being given. we may find,

25 The Suns declination D G.
26 The Suns amplitude D H.
27 The angle of the Suns position D G *H*.
by the 4 Problem 12 chap.

The Poles elevation G D H, and angle of the Suns position D *G H being given, we may find,*

28 The Suns declination D G.
29 The Suns amplitude D *H*.
39 The Suns altitude G *H*.
by the 5 Prob. 12 chap.

CHAP. XVII.

Of the Azimuthal Triangle.

THe fourth Right angled Spherical Triangle, I call an Azimuthal triangle, because the hypotenusa doth cut the Horizon, in the *East* and *West* Azimuths, and is represented by the triangle M D K, in which D K doth represent an arch of the Quadrant of altitude upon the Globe, the leg M K an arch of a Meridian, and M D an arch of the Equator, so that in this triangle, the five circular parts are,

Fig. 7

1 The hypotenusa D K, or arch of the Sun or Stars altitude being *East* or *West*.

2 The leg M K or declination of the Sun or Star.

3 The leg MD or right ascension of the Sun or Star.

4 The oblique angle M D K or angle of the Poles elevation above the Horizon.

5 The oblique angle M K D or angle of the Suns position.

And in this triangle the sides and oblique angle M D K are measured by inspection, and the quantity of the oblique angle M K D may be readily enough found, as hath been shewed in the 12 chap.

The several Problems in this triangle, are as followeth.

The Right ascension of the Sun or Star D M, and declination M K being given, we may find,

1 The sun or stars altitude D K being *East* or *West*.
2 The Poles elevation M D K.
3 The suns or stars angle of position M K D.
by the 1 *Probl.* 12 *chap.*

The

The Right ascension of the sun or star D M, and altitude D K being given, we may find,

4 The sun or stars declination M K
5 The angle of the Poles elevation M D K.
6 The angle of the sun or stars position M K D.
by the 1 Probl. of the 12 chap.

The declination of the sun or star M K, and the altitude D K being given, we may find,

7 The sun or stars Right ascension D M.
8 The angle of the Poles elevation M D K.
9 The angle of the sun or stars position M K D
by the 2 probl. of the 12 chap.

The sun or stars altitude D K, and angle of the poles elevation M D K being given, we may find,

10 The sun or stars declination M K
11 The sun or stars Right ascension M D.
12 The sun or stars angle of position M K D.
by the 3 Probl. of the 12 chap.

The sun or stars altitude D K, and angle of position M K D being given, we may find,

13 The suns or stars declination K M.
14 The suns or stars Right ascension D M.
15 The angle of the Poles elevation M D K.
by the 3 of the 12.

The sun or stars Right ascension D M, and angle of position M K D being given, we may find,

16 The sun or stars declination MK.
17 The sun or stars altitude DK.
18 Angle of the Poles elevation M D K.
by the 4 probl. of the 12 chap.

The

The sun or stars Right ascension D M, and angle of the Poles elevation M D K being given, we may find,

19 The sun or stars declination M K
20 The sun or stars altitude D K.
21 The angle of position D K M.
by the 4 probl. of the 12 chapt.

The sun or stars declination M K, & angle of position M K D being given, we may find,

22 The sun or stars Right ascension D M.
23 The sun or stars altitude D K.
24 The angle of the Poles elevation M D K.
by the 4 Probl. of the 12 chap.

The sun or stars declination M K, and angle of the Poles elevation M D K being given, we may find,

25 The sun or stars Right ascension M D.
26 The sun or stars altitude D K.
27 The angle of position D K M.
by the 4 Probl. of the 12 chap.

The angles of the Poles elevation M D K, and of position D K M being given, we may find,

28 The sun or stars declination M K.
29 The sun or stars Right ascension D M.
30 The sun or stars altitude D K.
by the 5 Probl. of the 12. chap.

CHAP.

CHAP. XVIII.

Of the Horizontal Triangle.

THe fist and last Right angled Spherical triangle that in this Treatise shall be mentioned, I call an Horizontal triangle, the hypotenusa thereof being an arch of the Horizon, and this is represented upon the Globe by D O L, in which the five circular parts are *Fig.* 7.

1 The hypotenusa and arch of the Horizon, or amplitude of the Sun at his rising or setting D O.

2 The sun or stars declination L O.

3 The ascentional difference D L, that is, the difference between, F L the Right ascension, and F D the oblique ascension.

4 The oblique angle of the Horizon and Equator, or height of the Equator L D O.

5 The angle of the Horizon and Meridian D O L.

And in this triangle the sides and oblique angle L D O are measured by inspection, and the quantity of the other oblique angle D O L, may be measured as hath been shewed in the 12 chapter.

The several Problems in this Triangle, are as followeth.

The sun or stars declination O L, and Ascensional difference D L being given, we may find,

1 The amplitude of their rising or setting D O.

2 The angle of the Horizon and Equator L D O.

3 The angle of the Horizon and meridian D O L.

by the 1 *probl. of the* 12 *chap.*

The ascensional difference of the sun or star D L, and amplitude D C being given, we may find,

4 The sun or stars declination O L.
5 The angle of the Horizon and Equator L D O.
6 The angle of the Horizon and Meridian D O L.
by the 2 probl. of the 12 chap.

The suns amplitude D O, and his present declination O L being given, we may find,

7 The ascensional difference D L.
8 The angle of the height of the Equator L D O.
9 The angle of the Horizon and Meridian D O L.
by the 2 probl. of the 12 chap.

The suns amplitude D O, and height of the Equator L D O being given, we may find,

10 The ascensional difference D L.
11 The suns present declination L O.
12 The angle of the Horizon and Meridian D O L.
by the 3 probl. of the 12 chap.

The suns amplitude D O, and angle of the Horizon and Meridian L O D being given, we may find,

13 The ascensional difference D L.
14 The suns present declination O L.
15 The angle of the height of the Equator O D L.
by the 3 probl. of the 12 chap.

The ascensional difference D L, and height of the Equator L D O being given, we may find,

16 The suns amplitude D O.
17 The suns present declination L O.
18 The angle of the Horizon and Meridian D O L.
by the 4 probl. of the 12 chap.

The

The ascensional difference D L, and angle of the Horizon and Meridian D O L being given, we may find,

19 The suns amplitude D C.
20 The suns present declination L O.
21 The height of the Equator L D O.
by the 4 probl. of the 12 chap.

The suns present declination L O, and angle of the Horizon and Meridian D O L being given, we may find,

22 The suns amplitude D O.
23 The ascensional difference D L.
24 The height of the Equator L D O.
by the 4 probl. of the 12 chap.

The suns present declination L O, and height of the Equator L D O being given, we may find,

25 The suns amplitude D O.
26 The ascensional difference D L.
27 The angle of the Horizon and Meridian D O L.
by the 4 probl. of the 12 chap.

The height of the Equator L D O, and angle of the Horizon and Meridian D O L being given, we may find,

28 The suns amplitude D O.
29 The suns present declination L O.
30 The ascensional difference D L
by the 5 probl. of the 12 chap.

By help of this triangle, besides those 30 Questions, we may find the Diurnal and Nocturnal arches of the Sun in any given latitude.

The Ascensional difference of the Sun being found, as hath been already shewed, you must add it to the Semidiurnal arch of the Right Sphere, which is 90, when the Sun is in the Northern Signes, and subtract it in the Southern, the sum or difference will be the

 Semi-

Semidiurnal arch, which doubled is the Day arch, whose complement to 24 hours is the Night arch, which bisected, is the time of the Suns rising.

As when the Sun is in the point C, the Ascentional difference D L must be added to the Semidiurnal arch of the Right Sphere Æ D, and there aggregate Æ L, is the Semidiurnal arch inquired, answereable to C M half the arch of the sun or stars motion for that day.

But when the place of the sun or star is in the Southern Signes, the Ascensional difference must be subtracted from Æ D, the remainer shall be the Semidiurnal arch of the sun or stars, as before.

Hence also by comparing the Ascensional difference with the Right ascension, you may find the oblique ascension and descension of the sun or stars.

In the former Diagram F L representeth the Right ascension, D L the ascensional difference, and F D the oblique ascension, now then if the declination of the sun or star be *North*.

Subtract the ascensional difference from the Right ascension, and the remainer is the oblique ascension.

Add the ascensional difference to the Right ascension and their sum is the oblique descension.

But if the declination of the sun or star be *South*, add the ascensional difference to the Right ascension, and their sum is the oblique ascension.

Subtract it from the Right ascension, the remainer is the Oblique descension.

And if you deduct the Oblique ascension of the sun from the Oblique ascension of any star, their difference being converted into time by allowing 15 degrees to every hour, and added to the time of the suns rise, will give the time of the day in which that star doth rise.

And the Oblique descension of the sun being subtracted from the Oblique descension of the star, their difference being converted into time as before, and added

added to the time of sun set, gives the time of the setting of that star.

Or if the place of the Sun or other Star be given, their Right and Oblique ascensions and descensions with their Semidiurnal and Seminocturnal arches, and time of their rising and setting in any latitude may be more easily found in this manner.

Bring the place of the sun or star to the Meridian above the Horizon, and the number of degrees comprehended between the Meridian, and vernal Æquinox or first point of *Aries* is the Right ascension: or to the Meridian under the Horizon, and the degree of the Equator that comes to the Meridian with it, is the Right descension.

For the oblique ascension & descension.

Bring the place of the sun or star to the *East* side of the Horizon, and the degree of the Equator cut by the Horizon, is the degree of oblique ascension of the sun or star: Or to the *West* side of the Horizon, and the degree of the Equinoctial cut by it, is the degree of Oblique descension.

For the Semidiurnal and Seminocturnal Arches.

Bring the place of the Sun to the Meridian above the Horizon, and then set the Index of the hour-circle, to the hour of 12, this done, bring the place of the Sun to the *East* side of the Horizon, and the hour Index will shew the hour from 12, which is the Semidiurnal arch, and time of his rising: or if you bring the place of the Sun to the *West* side of the Horizon, the hour Index will give the Semidiurnal arch again, and time of sun set, whose complement unto 12 is the Seminocturnal arch.

In like manner, the Globe being rectified by the place of the sun, as hath been said, bring the star whose Semidiurnal and Seminocturnal arches you desire with his time of rising and setting to the *East* or *West* of the Horizon, and you shall have your desire as before.

CHAP. XIX.
Of the Complemental Triangle.

HItherto we have spoken of the Right angled spherical triangles onely, & now we are come to the Oblique, the first whereof I call the Complemental triangle, for that all his sides be complements, *viz.* the complement of latitude or of the Poles elevation, the complement of the suns declination, and the complement of the suns altitude above the Horizon, commonly called his Almicanter; and this is represented upon the Globe, by an arch of the brass Meridian without the Globe, an arch of the great Meridian upon the Globle, and an arch of the Quadrant of altitude, as in the triangle AEZ, whose circular parts are,

Fig. 7.
1 Complement of the Poles elevation Z A.
2 Complement of the suns declination A E.
3 Complement of the suns altitude Z E.
4 The suns azimuth or distance from the *North* part of the Meridian E Z A.
5 The hour of the day or distance of the sun from 12 of the clock E A Z.
6 The angle of the suns position A E Z.

And in this triangle the quantity of each side is given by inspection, the arch of the Horizon B P is the measure of the angle A Z E, and the arch of the Equator Æ F is the measure of the angle E A Z and the angle A E Z be measured, as hath been shewed in the 13 Chapter.

The

The ſeveral Problems in this Triangle are as followeth.

By the 1 Problem of oblique angled ſpherical triangles.

The complement of the Poles elevation Z A, the complement of the ſuns declination A E, and the complement of the ſuns altitude Z E being given, we may find,

1 The ſuns azimuth from the *North* or ang. A Z E.
2 The ſuns diſtance from the Meridian Z A E.
3 The angle of the ſuns poſition A E Z.

By the 2 Problem of oblique angled ſpherical triangles.

The complement of the Poles elevation Z A, the complement of the ſuns altitudt Z E, and the ſuns azimuth from the North *A Z E being given, we may find,*

4 The diſtance from the Meridian Z A E.
5 The ſuns angle of poſition A E Z.
6 The complement of the ſuns declination A E.

The complement of the Poles elevation A Z, and the complement of the ſuns declination A E, and the ſuns diſtance from the Meridian Z A E being given, we may find,

7 The ſuns azimuth from the *North* A Z E.
8 The complement of the ſuns altitude Z E.
9 The angle of the ſuns poſition A E Z.

The complement of the ſuns declination A E, the angle of the Suns poſition A E Z, and the complement of the ſuns altitude E Z being given, we may find,

10 The ſuns azimuth from the *North* A Z E.
11 The complement of the Poles elevation A Z.
12 The ſuns diſtance from the Meridian E A Z.

By

By the 3 Problem of Oblique angled ſpherical triangles.

The ſuns angle of poſition A E Z, the complement of the ſuns altitude E Z, and the ſuns azimuth from the North *A Z E being given, we may find,*

13 The complement of the Poles elevation A Z.
14 The ſuns diſtance from the Meridian E A Z.
15 The complement of the ſuns declination A E.

The ſuns azimuth from the North *A Z E, the complement of the Poles elevation A Z, and the ſuns diſtance from the Meridian Z A E being given, we may find,*

16 The complement of the ſuns declination A E.
17 The ſuns angle of poſition A E Z.
16 The complement of the ſuns altitude E Z.

The ſuns diſtance from the Meridian E A Z, the complement of the ſuns declination A E, and the ſuns angle of poſition A E Z being given, we may find,

19 The complement of the ſuns altitude E Z.
20 The ſuns azimuth from the *North* A Z E.
21 The complement of the Poles elevation A Z.

By the 4 Problem of Oblique angled ſpherical triangles.

The complement of the Poles elevation A Z, the complement of the ſuns declination A E, and the ſuns angle of poſition A E Z being given, we may find,

22 The complement of the ſuns altitude E Z.
23 The ſuns azimuth from the *North* A Z E.
24 The ſuns diſtance from the Meridian E A Z.

The

The complement of the Poles elevation A Z, the complement of the suns declination A E, and the suns azimuth from the North A Z E being given, we may find,

25 The complement of the suns altitude E Z.
26 The suns angle of position A E Z.
27 The suns distance from the meridian E A Z.

The complement of the suns declination A E, and the complement of the suns altitude E Z, and the suns azimuth from the North *A Z E being given, we may find,*

28 The complement of the Poles elevation A Z.
29 The suns distance from the meridian E A Z.
30 The suns angle of position A E Z.

The complement of the suns declination A E, the complement of the suns altitude E Z, and the suns distance from the Meridian E A Z being given, we may find,

31 The complement of the Poles elevation A Z.
32 The suns azimuth from the *North* A Z E.
33 The suns angle of position A E Z.

The complement of the suns altitude E Z, the complement of the poles elevation A Z, and the suns distance from the Meridian E A Z being given, we may find,

34 The complement of the suns declination A E.
35 The suns angle of position A E Z.
36 The suus azimuth from the *North* A Z E.

The complement of the suns altitude E Z, the complement of the Poles elevation A Z, and the suns angle of position A E Z being given, we may find,

37 The complement of the suns declination A E.
38 The suns distance from the meridian E A Z.
39 The suns azimuth from the *North* A Z E.

By the fifth Problem of Oblique angled spherical triangles.

1 *The suns azimuth from the* North *A Z E, the suns distance from the Meridian E A Z, and the complement of the suns declination A E, being given, we may find,*

40 The complement of the poles elevation A Z.
41 The complement of the suns altitude E Z.
42 The suns angle of position A E Z.

2 *The suns azimuth from the* North *A Z E, the suns distance from the Meridian E A Z, and the complement of the suns altitude E Z being given, we may find,*

43 The complement of the poles elevation A Z.
44 The complement of the suns declination A E.
45 The suns angle of position A E Z.

3 *The suns distance from the Meridian E A Z, the suns angle of position A E Z, and the complement of the suns altitude E Z being given, we may find,*

46 The complement of the poles elevation A Z.
47 The complement of the suns declination A E.
48 The suns azimuth from the *North* A Z E.

4 *The suns distance from the Meridian E A Z, the suns angle of position A E Z, and the complement of the poles elevation A Z being given, we may find,*

49 The complement of the suns altitude E Z.
50 The complement of suns declination A E.
51 The suns azimuth from the *North* A Z E.

5 *The suns angle of position A E Z, the suns azimuth from the* North *A Z E, and the complement of the poles elevation A Z being given, we may find,*

52 The complement of the suns altitude E Z.
53 The complement of the suns declination A E.
54 The suns distance from the meridian E A Z.

6 *The*

The suns angle of position A E Z, the suns azimuth from the North *A Z E, and the complement of the suns declination A E being given, we may find,*

55 The complement of the suns altitude E Z.
56 The complement of the poles elevation A Z.
57 The suns distance from the meridian E A Z.

By the sixth Problem of oblique angled spherical triangles.

The suns angle of position A E Z, the suns azimuth from the North *A Z E, and suns distance from the Meridian E A Z being given, we may find,*

58 The complement of the suns altitude E Z.
59 The complement of the poles elevation A Z.
60 The complement of the suns declination A E.

CHAP XX.

Of the Geographical Triangle.

THe second oblique angled spherical triangle, I call a Geographical or Nautical triangle, because it serveth to resolve those Problems which concern Geography and Navigation and this is also represented by the triangle A E Z, whose Circular parts, are

1. The complement of latitude belonging to that Town or City represented by the Zenith point at Z, or side A Z.

2 The distance between the two places at Z and E, or the side E Z.

3 The complement of the latitude of the place at E, or the side A E.

4 The difference of longitude between the two places at E and Z, or the angle E A Z.

5 The point of the compaſs leading from Z to E, or the angle A Z E.

6 The point of the Compaſs leading from E to Z, or the angle of poſition A E Z.

¶ And in this triangle the quantity of each ſide and angle is given, as hath been ſaid in the laſt chap.

The ſeveral Problems of this Triangle, are

By the firſt Problem of oblique angled ſpherical triangles.

The complement of the latitude of one place A Z, the complement of the latitude of the other place A E, and the diſtance between them E Z being given, we may find,

1 Their difference of longitude E A Z.

2 The point of the compaſs leading from Z to E, or A Z E.

3 The angle of poſition A E Z.

By the ſecond Problem of oblique angled ſpherical triangles.

1 *The diſtance between the two places E Z, the courſe or bearing from Z to E, or angle E Z A, and the ſide A Z, or complement of the latitude of the place at Z being given, we may find.*

4 The angle of poſition A E Z.

5 The complement of latitude of the place at E, or A E.

6 Their difference of longitude E A Z.

2 *The ſide A Z or the complement of the latitude of Z, the ſide A E or complement of the latitude of E, and their difference of longitude E A Z being given, we may find,*

7 The angle of poſition A E Z.

8 The diſtance between the two places E Z.

9 The angle E Z A, or bearing from Z to E.

3 The

3 *The side* A E *or complement of the latitude of* E, *the angle of position* A E Z, *and the distance between the two places* E Z *being given, we may find,*

10 The bearing from Z to E, or angle E Z A.
11 The side A Z or complement of the latitude of Z.
12 The difference of longitude E A Z.

By the third Problem of Oblique angled spherical triangles.

1 *The angle of position* A E Z.

The side E Z *or distance between the two places, and the angle* E Z A, *or bearing from* Z *to* E *being given, we may find,*

13 The side A Z or complem. of the latitude of Z
14 The difference of longitude E A Z.
15 The side A E or complem. of the latitude of E.

2 *The bearing from* Z *to* E, *or angle* A Z E, *the side* A Z, *or complement of the latitude of* Z, *and the difference of longitude* E Z A *being given, we may find,*

16 The side A E or compl. of latitude of E.
17 The angle A E Z.
18 The distance between them E Z.

3 *The difference of longitude* E A Z, *the side* A E *or complement of latitude of* E, *and the bearing from* E *to* Z, *or angle* A E Z *being given, we may find,*

19 The distance between the two places E Z.
20 The angle A Z E or bearing from Z to E.
21 The side A Z or compl. of latitude of Z.

By the fourth Problem of oblique angled spherical triangles.

1 *The side* A Z *or co-latitude of the place at* Z, *the side* Z E *or co-latitude of the place at* E, *and the angle* A E Z, *or bearing from* E *to* Z *being given, we may find,*

22 The side E Z or distance between the two places.
23 The angle A Z E or bearing from Z to E.
24 The angle E A Z or difference of longitude.

2 *The side* A Z *or co-latitude of* Z, *the side* A E *or co-latitude of* E, *and the angle* A Z E *or bearing from* Z *to* E *being given, we may find,*

25 The side E Z or distance between the 2 places.
26 The angle A E Z or bearing from E to Z
27 The angle E A Z or difference of longitude.

3 *The side* A E *or latitude of* E, *the side* E Z *or distance between the two places, and the angle* A Z E *or bearing from* Z *to* E *being given, we may find,*

28 The side A Z or co-latitude of Z.
29 The angle E A Z or difference of longitude.
30 The angle A E Z or bearing from E to Z.

4 *The side* A E *or co-latitude of* E, *the side* E Z *or distance between the two places, and the angle* E A Z *or difference of longitude being given, we may find,*

31 The side A Z or co-latitude of Z.
23 The angle A Z E or bearing from Z to E.
33 The angle A E Z or bearing from E to Z.

5 *The side* E Z *or distance between the places, the side* A Z *or co-latitude of* Z, *and the angle* E A Z *or difference of longitude being given, we may find,*

34 The side A E or co-latitude of E.
35 The angle A E Z or bearing from E to Z.
36 The angle A Z E or bearing from Z E.

6 The

6 The side E Z or distance between the places, the side A Z or co-latitude of Z, and the angle A E Z or bearing from E to Z being given, we may find,

37 The side A E or co-latitude of E.
38 The angle E A Z or difference of longitude.
39 The angle A Z E or bearing from Z to E.

By the fifth Problem of Oblique angled spherical triangles.

1 The angle A Z E or bearing from Z to E the angle E A Z or difference of longitude, and the side A E or co-latitude of E being given, we may find,

40 The side A z or co-latitude of Z.
41 The side E z or distance between the places.
42 The angle A E z or bearing from E to z.

2 The angle A Z E or bearing from Z to E, the angle E A Z or difference of longitude, and the side E Z or distance between the places being given, we may find,

43 The side A z or co-latitude of Z.
44 The side A E or co-latitude of E.
45 The angle A E z or bearing from E to Z.

3 The angle E A Z or difference of longitude, the angle A E Z or bearing from E to Z, & the side E Z or distance between the two places being given, we may find,

46 The side A Z or co latitude of Z.
47 The side A E or co-latitude of E.
48 The angle A Z E or bearing from Z to E.

4 The angle E A z or difference of longitude, the angle A E Z or bearing from E to z, and the side A z or co-latitude of Z being given, we may find,

49 The side E Z or distance between the 2 places.
50 The side A E or co-latitude of E.
51 The angle A z E or bearing from z to E.

The

5 *The angle A E Z or bearing from E to Z, the angle A Z E or bearing from Z to E, and the side A Z or latitude of Z, being given, we may find,*

52 The side E Z or distance between the places.
53 The side A E or co-latitude of E.
54 The angle E A Z or difference of longitude.

6 *The angle E A Z or bearing from E to Z, the angle A Z E or bearing from Z to E, and the side A E or co-latitude of E being given, we may find,*

55 The side E Z or distance between the places.
56 The side A Z or co-latitude of Z.
57 The angle E A Z or difference of longitude.

By the sixth Problem of oblique angled Sperical triangles.

The angle E A Z or difference of longitude, the angle A Z E or bearing from Z to E, and the angle A E Z or bearing from E to Z being given, we may find,

58 The side E Z or distance between the places.
59 The side A Z or co-latitude of Z.
60 The side A E or co-latitude of E.

CHAP. XXI.

Of the Polar Triangle.

THE third Oblique angled Spherical Triangle, is called a Polar Triangle, because one side thereof is the distance between the Poles of the World, and the Poles of the Equinoctial, the other sides are the arches of a Meridian, and a Circle of longitude.

The

This Triangle is represented upon the Cœlestial Globe, by the Triangle A Z E in which the circular parts, are *Fig. 8.*

1 The arch A Z, the distance between the Pole of the world, and the pole of the ecliptick.

2 The arch A E the complement of F E the declination of a star at E.

3 The arch Z E the complement of B E the stars Nothern latitude from the ecliptick.

4 The complement of the stars Right ascension, or the angle E A Z whose measure in the Æquinoctial is the arch Æ F.

5 The angle of the stars longitude A Z E.

6 The angle of the stars position A E Z.

¶ And in this Triangle the quantity of each side and angle is given, as hath been shewed *Chap.* 19.

The several Problems in this Triangle, are

By the first Problem of Oblique angled Spherical Triangles.

The distance between the Pole of the World, and the Pole of the ecliptick A Z, the complement of declination A E, and the complement of latitude Z E being given, we may find,

1 The angle of the stars Right Ascension E A Z.
2 The angle of the stars longitude A Z E.
3 The angle of the stars position A E Z.

By the second Problem of oblique angled Spherical Triangles.

1 *The complement of latitude E Z, the angle of the stars longitude E Z A, and the distance between the Pole of the World, and the Pole of the ecliptick, or the suns greatest declination Z A being given, we may find,*

4 The stars angle of position A E Z.
5 The complement of the stars declination A E.
6 The complement of the stars R. ascension E A Z.

2 *The side AZ or the suns greatest declination, the side AE or complement of the stars declination, & the angle EAZ or complement of the stars Right ascension being given, we may find,*

7 The angle of the stars position A E Z.
8 The complement of the stars latitude E Z.
9 The angle of the stars longitude E Z A.

3 *The complement of the stars declination A E, the complement of the stars latitude E Z, and the stars angle of position A E Z being given, we may find,*

10 The angle of the stars longitude E A Z.
11 The suns greatest declination Z A.
12 The compl. of the stars R. ascension E A Z.

By the third Problem of oblique angled Spherical Triangles.

1 *The angle of the stars position A E Z, the complement of the stars latitude E Z, and the angle of the stars longitude being given, we may find,*

13 The suns greatest declination Z A.
14 The complement of the stars R. A. E A Z.
15 The complement of the stars declination A E.

2 *The angle of the stars longitude E Z A, the suns greatest declination Z A, and the complement of the stars R. ascension E A Z being given, we may find,*

16 The complement of the stars declination A E.
17 The angle of the stars position A E Z.
18 The complement of the stars latitude E Z.

3 *The complement of the stars R. ascension E A Z, the complement of the stars declination A E, and the angle of the stars position being given, we may find,*

19 The complement of the stars latitude E Z.

20 The

20 The angle of the stars longitude E Z A.
21 The suns greatest declination Z A.

By the fourth Problem of oblique angled Spherical Triangles.

1 *The suns greatest declination A Z, the complement of the stars declination A* E, *and the stars angle of position A Z* E *being given, we may find,*

22 The complement of the stars latitude E Z.
23 The angle of the stars longitude E Z A.
24 The compl. of the stars R. ascension E A Z.

2 *The suns greatest declination Z A, the complement of the stars declination A E, and the angle of the stars longitude* E Z *A being given, we may find,*

25 The complement of the stars latitude E Z.
26 The angle of the stars position A E Z.
27 The compl. of the stars R. ascension E A Z.

3 *The complement of the stars declination* A E, *the complement of the stars latitude* E Z, *and the angle of the stars longitude* E Z A *being given, we may find,*

28 The suns greatest declination A Z.
29 The compl. of the stars R. ascension E A Z.
30 The angle of the stars position A E Z.

4 *The compl. of the stars declination* A E, *the compl. of the stars latitude* E Z, *and the compl. of the stars R. ascension* E *A Z being given, we may find,*

31 The suns greatest declination A Z.
32 The angle of the stars longitude E Z A:
33 The angle of the stars position A E Z.

5 *The compl. of the stars latitude E Z, the suns greatest declination A E, and the compl. of the stars R ascension E A Z being given, we may find,*

34 The complement of the stars declination A E.
35 The angle of the stars position A E Z.
36 The angle of the stars longitude E Z A.

6 *The compl. of the stars latitude E Z, the suns greatest declination A Z, and the stars angle of position A E Z being given, we may find,*

37 The compl. of the stars declination A E.
38 The compl. of the stars R. ascension E A Z.
39 The angle of the stars longitude E Z A.

By the fifth Problem of oblique angled Spherical Triangles.

1 *The angle of the stars longitude E Z A, the compl. of the stars R. ascension E A Z, and the compl. of the stars declination A E being given, we may find,*

40 The suns greatest declin. A Z.
41 The compl. of the stars latitude E Z.
42 The angle of the stars position A E Z.

2 *The angle of the stars longitude E Z A, the compl. of the stars right ascension E A Z, and the compl. of the stars latitude E Z being given, we may find,*

43 The suns greatest declinat. A Z.
44 The compl. of the stars declinat. A E.
45 The angle of the stars position A E Z.

3 *The compl. of the stars R ascension E A Z, the angle of the stars position A E Z, and the compl. of the stars latitude E Z being given, we may find,*

46 The suns greatest declinat. A Z.
47 The compl. of the stars declinat. A E.
48 The angle of the stars longitude E Z A.

The

4 *The compl. of the stars R. ascension E A Z, the angle of the stars position A E Z, and the suns greatest declination A Z being given, we may find,*

49 The compl. of the stars latitude E Z.
50 The compl. of the stars declinat. A E.
51 The angle of the stars longitude E Z A.

5 *The angle of the stars position A E Z, the angle of the stars longitude E Z A, and the suns greatest declination A Z being given, we may find,*

52 The compl. of the stars latitude E Z.
53 The compl. of the stars declinat. A E.
54 The compl. of the stars R. ascension E A Z.

6 *The angle of the stars position A E Z, the angle of the stars longitude E Z A, and the compl. of the stars declination A E being given, we may find,*

55 The compl. of the stars latitude E Z.
56 The suns greatest declination A Z.
57 The compl. of the stars R. ascension E A Z.

By the sixth Problem of oblique angled Spherical Triangles.

The angles of the stars longitude E Z A, the compl. of the stars R. ascension E A Z, and the angle of the stars position A E Z, being given, we may find,

58 The suns greatest declination A Z.
59 The compl. of the stars declination Z E.
60 The compl. of the stars latitude E Z.

CHAP. XXII.

To let fall a perpendicular that ſhall divide any oblique angled ſpherical into two Reght.

IN the ſolution of oblique angled Spherical Triangles by the Canons of Sines and Tangents, this Problem is in ſeveral Caſes very uſeful: but in reſolving thoſe Caſes by the Globe, we have need of it, ſeeing all of them may otherwiſe be reſolved, as hath been already ſhewed; yet for variety ſake, we have inſerted this way alſo.

In the Polar Triangle A Z E, let there be given.

The ſides AZ, EZ and the ang. EAZ.

And let the ſide A E be inquired.

To reſolve this Caſe by the Catholick Propoſition of Right angled Spherical Triangles and the Canon of Sines and Tangents, a perpendicular may be let fall from Z upon the ſide inquired A E, the which upon the Globe may be done in this manner.

Where the arch of the Meridian or ſide A E (upon which the perpendicular is to fall) ſhall cut the Æquinoctial, make a mark, which in the Figure is at F: and from this point of interſection of the Meridian with the Æquinoctial at F, reckon a Quadrant or 90 degrees, which ſuppoſe to be at C, a thin plate of braſs with a nut at one end thereof, whereby to faſten it to the Meridian, as you doe the Quadrant of altitude, being graduated as that is, but of a larger extent, (for that a Quadrant in this Caſe will not ſuffice) being faſtened at Z, & turned about till it cut the point C in the Æqui-

Æquinoctial, will describe upon the Globe the arch of a great Circle Z H C, intersecting the side A E at right angles in the point H, upon this ground; the point C in the Æquinoctial is the Pole of the Circle A E F, now all great Circles which passing through the point C, shall intersect the Meridian A E F, will intersect it at Right angles *by the 12 of the 10 Chapter.*

Thus the Quadrant of altitude being fastened in the zenith of any place, which way soever you turne it, will cut the Horizon at Right angles, because the zenith is the Pole of the Horizon, and so here, because the point C in the Æquinoctial is the Pole of the Meridian A E F, therefore the arch of a great Circle Z H C must needs cut that Meridian at Right angles in the point H, & the oblique angled Spherical triangle AZE is divided into the two Right angled triangles EZH & AZH by the perpendicular ZH, the quantity whereof is given by inspection upon the graduated plate and the segments of the side A E, that is, A H and E H are both measured in the Meridian, so that in the triangle AHZ, the angle A Z H onely is unknown, which may be found by turning the Triangle as hath been shewed in the 12 Chapter.

And in the Triangle E H Z the three sides and Right angle at H are known, which are more than need be given to find the angles at E and Z, as hath been also shewed before: and thus we have (though breifely, yet plainly) shewed the solution of Spherical Triangles upon the Globe, not onely in the General, but in all the Problems which are conteined in eight particular Triangles, which doe amount to 330, and are sufficient, sure (if well understood) to inform the Reader, by what meanes such other Problems may be best resolved, which are not here expressed.

FINIS.

Fig: 7.

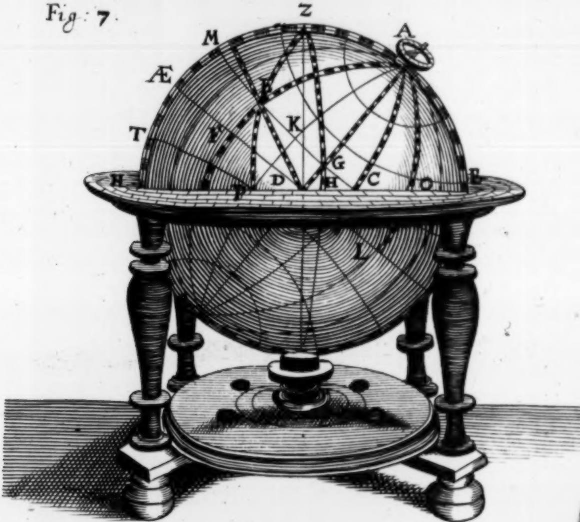
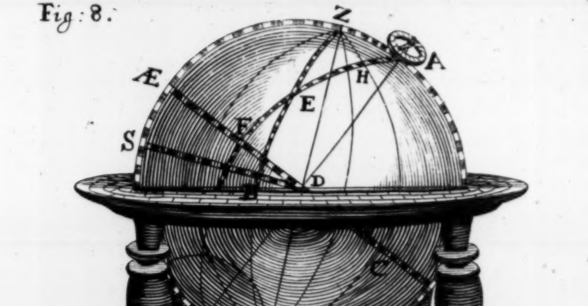


Fig: 8.



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THE PROJECTION OF THE SPHERE, OR DELINEATION OF THE GLOBE,

Upon the Plane of any great Circle, according to the Laws of the Stereographick or Circular projection.

By JOHN NEWTON. *M. A.*

LONDON,

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THE PROJECTION OF THE SPHERE.

CHAP. I.

To project the Circles of the Globe in plano according to their several positions in respect of the eye.

A Sphære or Globe which is one of the best Instruments for informing the phansie, is one of the worst for resolving particular Questions; & therefore for that purpose other Instruments have been invented, amongst which the *Mathematical Jewel* hath not been least in estimation, and yet even that is so perplexed, with a multiplicity of lines that the resolution of most Questions by that Instrument is very troublesome, this trouble can no way be better avoided (if there presentation of the Sphære and resolution of the Question be

both required at one time) then by a particular delineation of thoſe lines, which in the Queſtion propounded, are both given and required.

How that may be done upon thoſe grounds and principles by which the *Jewel* is made, is that which is intended by this preſent diſcourſe.

And that excellent Inſtrument now known by the name of *Blagraves Mathematical Jewel*, doth depend upon that projection of the Sphere, which we call the *Steriographick* or Circular projection, concerning which that this diſcourſe, though ſhort, may yet be plain, it will be neceſſary to declare.

1 How the Circles of the Sphere or Globe both great and ſmall may in all their ſeveral poſitions and ſituations according to the Laws of this projection be deſcribed in plano.

2 How any arch of thoſe circles ſo projected may be meaſured, either in reference to it ſelf or in reference to the primitive Circle.

And 3 How an angle propounded may be projected or being projected may be meaſured.

In the firſt of theſe, the projection of any Circle of the Sphere in any poſition or ſituation, we are to conſider: firſt, the ſeveral varieties of poſition that the circles of the Sphere may have in reſpect of that point from whence the eye beholdeth them, and then the Problems by which they may be projected in theſe poſitions.

The varieties of poſition are but three, for all circles of the Sphere are ſeen by the eye, either in a *Perpendicular*, in a *Direct*, or in an *Oblique Aſpect*.

1 A Circle is then ſaid to be *Perpendicular* to the ſight, when the ſuperficies of it is paralle to it, as ſuppoſe the eye to be placed in any part of the edge of a round Table, and thence to behold the whole ſuperficies of that circular plain, then is the ſuperficies of that circle parallel, and the circle it ſelf perpendicular to the ſight.

2 A Circle is then said to have a *Direct Aspect* to the eye, when the superficies of it, is directly opposite to it, intersecting the supposed line from the eye to the center of the Circle at Right angles.

3 A Circle is then said to have an *Oblique Aspect* to the eye, when the superficies of it, is neither parallel with, nor opposite to, the visual line, but cuteth it at oblique angles. How a circle seen by the eye in any of those positions may be projected in plano shall be explain'd in the Problems following.

PROBLEM 1.

To describe a Circle upon a plane, which is perpendicular to the sight; and to measure the parts thereof being so described.

A circle in this position being projected, will become a straight line and may upon either hand be infinitely extended, as by Example will better appear, then can be exprest by many words. *Fig.* 1.

Let A B C D be such a Circle, in which let the eye be placed at A, the Diameter B D being extended both wayes as far as you please, shall be the projection of this Circle; and may be distinguished into pars in this manner, upon your Compasses to what extent you please, and thereby divide the given Circle into any number of parts, suppose 8 by the points at A. B. I. C. K. D. and L. lines drawn from A through these points shall cut the extended Diameter B D in the points F. B. O. E. P. D. G., and the straight lines F E. B E. and O E. shall be the projection of the arches H C. B C. and I C. and these arches are the double measures of those lines being the double measures of the angles at A, and E F being a Tangent line answerable to the radius A E which is the Radius of the Circle by which these angles are measured the lines E F. E B. and E O. are the tangents of these angles F A E. B A E. & O A E

now

now because the Tangent of 90 deg. cannot be limited, therefore a Circle thus projected will become an infinite line.

PROBLEM 2.

To describe a Circle upon a plane, which hath a Direct Aspect to the eye, and to measure the parts thereof, being so described.

Fig. 1. A Circle in this position being projected will be a perfect Circle, as if the pole of the World in a Sphere or Globe be elevated into the Zenith, and the eye there posited, all the Circles between the Pole & the Equinoctial are in this position, and also parallel to one another, now all their Centers meeting in one Axis or Radius, being projected will all meet in one point, having therefore made a primitive Circle ABCD, if you open your Compasses to any other extent as to the extent of the Radius EM a circle drawn upon the point E, shall be the projection of that circle so posited, the parts whereof are correspondent to the parts of the primitive circle, that is, the arch MN in the small circle doth contein as many degrees as the arch IC in the great, and so of any other.

PROBLEM 3.

Two points in a Circle being given, to find the center of an arch of another Circle, which shall pass through the two given points, and also devide the given Circle into two equal parts.

Fig. 2. IN the circle HFDG, let there be given the two points B and C through one of the points as B, draw the Diameter LM, which let be extended at pleasure, crosse this Diameter at Right angles with the Diameter DR, and draw the line BD and make ED per-

perpendicular to B D in the point D, where the line D E shall intersect the Diameter L M, which in this figure is in the point E, shall be the third point, by which to draw the arch F B C E, the center whereof may be found by the 6 Proposition of the 3 chapter of the first Part. A circle drawn according to these directions shall pass through the given points, and also divide the given circle into two equal parts.

DEMONSTATION.

As. A B. A D :: A D. A E. because the Triangles A B D and A B E are like *by the* 8 *of the* 6 *of Euclide*, and A G continued will fall upon F: If not let A G continued fall upon H, and continue G A H to K, then shall it be A B. A G :: A K. A E *by the* 35 *of the* 3 *Eucl.* And the Rectangle of A G × A K = A D × A D = A G × A F = A G × A H and by consequence A H equal to A K which is absurd.

PROBLEM 4.

To describe a Circle upon a plain which hath an Oblique Aspect to the eye, and to measure the parts thereof being so described.

A Circle in this position being projected will be a perfect circle, in the Description whereof the greatest difficulty is in finding the center, for which sometimes there are but two, sometimes there are three points given; when two points are onely given, the third must be found, as hath been shewed in the last Problem; how to bring three points into a circle you may see in the 3 *chap. of the* 1 *Part*: but if the distance between the first and second points be equal to the distance between the second and the third, the centers of these circles may be more easily found, by these directions following.

Fig. 3. In the Circle B C D E, let the line of measure be B A B, in whose lower pole E, let the eye be placed, let G A F be the great circle to be described, whose obliquity is B G, and a line drawn from E to G, shall give the point H in the line of measures, and so you have the three points E. H. C. to draw this circle by: make D F equal to B G, and draw the line E F, which being continued till it cut the line of measures will limit the Diameter of that circle at B: make C L and L M each of them equal to B G, and draw the lines E L and E M, then shall N be the pole, and K the center of the circle E H C B. The Radius being H K = K E, for the angle K H E = K E H, because the angle H = A G H + G A E, that is, A E H + A E K, for the arch C M = 2 B G.

Thus by a line of Chords; By the line of Tangents and Secants the centers of these circles may be thus found, the Tangent of the circles obliquity set from A, shall give the center K in the line of measures, or the Secant thereof from H: because the half sum of the Tangent, and Tangent complement of an arch, is equal to the Secant of the difference of those arches. See *Briggs Trig. Brit.* and either of them equal to the radius of that circle.

To describe the lesser circles or parallels to the oblique circle E H C B, as suppose the parallel P S R do thus; Draw the Radius A P, and perpendicular thereunto at the point P, draw the line P Q, then shall Q in the line of measures be the center of the parallel P S R, the Radius being S Q = P Q, for the angle P S Q = S P Q, because the angle S = A P S + P A S; that is, S P R + R P Q. Or thus, the co-tangent of C P set from P to the line of measures will reach to Q the center of the parallel.

See *Briggs Trig. Brit.* first part 17 chap.

In this Scheme H C = B C = A C 90 degrees, whence it followeth, that the Quadrant H C is divided into degrees from its pole N by the degrees of the Quadrant B C, that is, a Ruler laid from N to any

part

part of the Quadrant B C, will cut as many degrees in the Quadrant H C as it doth in the Quadrant B C, and thus the arch H X = B T, the arch H y = B V and y X = V T: the equality of these arches is very plain, And on the contrary the given point X in the Quadrant H C may be measured in the limb, by drawing the line N X T from the pole thereof at R. And if in the same great circle E H C another point be given as at y, if through this point you draw the line N y V, the arch V T will be the measure of the space between y and X, as was required.

PROBLEM 5.

To describe a great Circle upon a plain, which hath an oblique Aspect to the eye, when the two extream points given are not equidistant from the third.

WHen in a great circle which hath an oblique aspect to the eye there are three points given, and the third point in the line of measures, we have besides the common way of bringing three points into a circle, shewed in the last Problem a more compendious way; and although the common way will also perform what is propounded in this Problem, yet may it be more neatly and expediously described in this manner.

Having drawn the primitive circle B C D E, and the two Diameters B A D, and C A E, let there be given the point F, by which to describe the circle C F E, though the point F, draw the Diameter Æ A æ, and the Diameter H A G at right angles thereunto, a ruler laid from G unto F, shall cut the primitive circle in K, and making K L = H K, a line drawn from G through L, will give N the center of the circle H M G in the Diameter Æ A æ extended, cutting the Diameter B A D in M, the pole of the circle C F E, and a Ruler laid from C unto M, will cut the primitive *Fig.* 4.

 circle

circle in P, and making P Q = P E a Ruler laid from C unto Q, will cut the Diameter B A D in R, the center of the circle C F E desired.

This done, the Radius of any other circle passing through the same point F is easily found, as suppose it were now required, to describe the circle V F M, draw Y Z at right angles to M V, a Ruler laid from G unto S, shall cut the primitive circle in T, make T X = H T, a Ruler laid from G unto X, shall cut the Diameter Æ A æ in W, make A O equal unto A W, so shall F W be Radius and the point O the center of the circle V F M as was desired.

6 PROBLEM.

A small circle which hath an oblique aspect to the eye being projected, to measure the arches of it.

Fig. 5. LEt P G F Q represent a smaller circle in an oblique aspect to the eye, and let the measure of the arch G F be required; perfect the circle P G F Q, and draw the lines G C H and F C K, I say the arch K H is the measure of G F, in the small circle, and may easily be reduced to the like arch in the primitive circle, for if you make M R equal to A C, and upon the point M, describe a circle at that extent of the compasses the arch S R T shall be equal to the measure of G F in the primitive circle B C D E.

7 PROBLEM

To project any angle propounded, or measure any angle that is projected.

IN this Problem there are four varieties, for first, an angle to be projected, may be conteined by two great circles which are perpendicular to the sight, that is, by two circles which being projected, will become straight lines, and meet in the center of the primitive circle, in this case you must proceed as in the protraction of right lined angles hath been shewed.

Secondly,

Fig: 1.

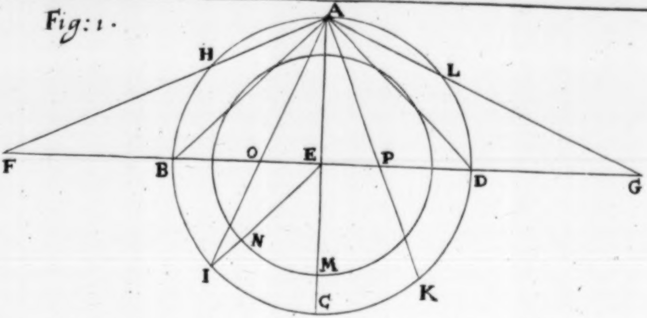


Fig: 2.

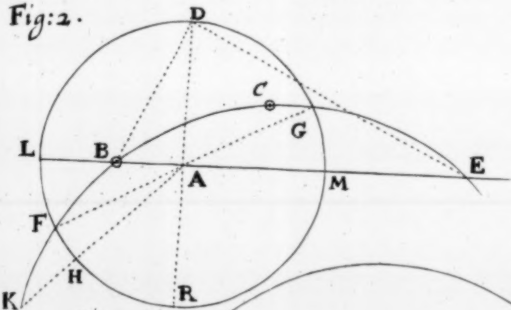
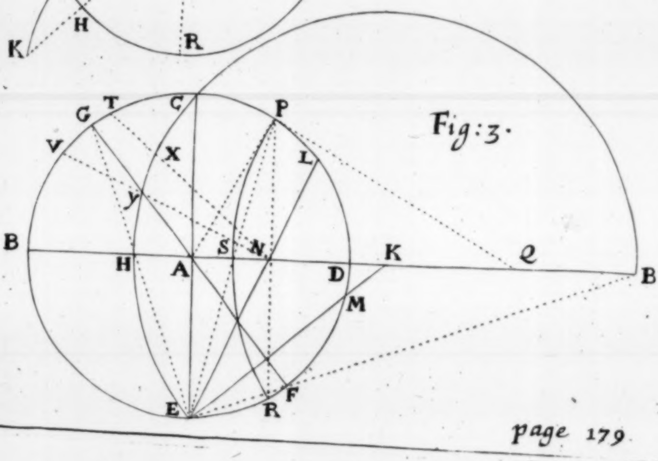


Fig: 3.



Secondly, the arches conteining the angle, may be one of them, an arch of the primitive circle, and the other an arch of an oblique, and then this method is to be used.

From the point assigned set off a Quadrant or 90 degrees, and draw two Diameters, as in the circle HZBX, let the point assigned be Z, a Quadrant from thence is H, the two Diameters ZDX & HDB, set the quantity of the angle given, by help of your line of chords from H unto G, a Ruler laid from Z unto G, will cut the Diameter HDB in the point P, so may you draw the circle ZPX, as hath been shewed, and then the angle HZP is equal to the arch HG, which is also equal to HP: if the angle had been projected and the measure required, a Ruler laid from Z unto P would cut the primitive circle in G, and HG would be the measure of it. *Fig. 6.*

Thirdly, The arches conteining to be projected may be one of them the arch of a circle perpendicular to the sight and the other oblique, as the angle ÆRZ or XRæ, and then the angle propounded may be thus projected; a Ruler laid from F the pole of the circle ZPX unto R, the angular point propounded will cut the primitive circle in L, make XN equal to HL, a Ruler laid from F unto N, will cut the circle ZPX in E, and a Ruler laid from R unto E, will cut the primitive circle in V, make Væ equal to the angle propounded, & draw the Diameter ÆDæ then is the angle XRæ equal to the arch Væ as was required. If the angle had been projected and the measure required, a Ruler laid from F to N would give E, from R to E would give V, and Væ should be the measure of the angle. *Fig. 7.*

Fourthly, The arches conteining the angle to be projected, may be both of them arches of the oblique circles, as the angle RÆZ or XÆN, and in this case the projecting, or measuring of an angle projected, is but little differing from the last variety for a ruler laid from F the pole of the circle ZPX unto Æ, the

angular point propounded, will cut the primitive circle in I, make X W equal unto H I: a Ruler laid from F unto W, will cut the circle Z P X in *α*, and a Ruler laid from Æ unto *α*, will cut the primitive circle in *a*, make *a b* equal to the angle propounded, a Ruler laid from Æ unto *b*, will cut the Diameter H B in F, and *by the 3 Probl. of the 1 chapter*, having the two points Æ and F given, you may describe the circle R Æ F N, which is the onely difference between the projecting of this and the former angles, for there a Ruler laid from *α* unto R, did cut the Diameter H B in D the center of the primitive circle, and so the other circle to be projected became a straight line, as it ought, the angle being propounded to be conteined by two circles one oblique the other perpendicular to the sight; but both the circles were to be oblique in this variety, and therefore a Ruler laid from *b* unto Æ must not cut the Diameter H B in D: but some other point, as here at F by help whereof the circle R Æ F N must be described as hath been said. To project this angle is seldome propounded, but this angle being projected the measure thereof is often required; which may be thus effected.

Fig. 8. Having described the circle M *a* W by the directions of the 5 Problem, which must pass through *a* and *n* the poles of the circles Z E X and A E S, a Ruler laid from E to the intersections of that circle, with the other at *f* and *g*, will cut the primitive circle in *h* and *l*, the measure of the angle at E.

Or, a Ruler laid from E to *a* and *n*, will cut the primitive circle in *m* and *q* equal to *h l* the measure of the angle A E Z, as before.

CHAP.

CHAP. II.

Of the Solution of Right angled Spherical Triangles by the Stereographick projection.

IN Right angled Spherical Triangles, there are as (hath been ſaid in the 12 chapter of the ſecond Part hereof) but five things which come into queſtion, the three ſides and the two acute angles; becauſe the third being a Right angle or 90 degrees is alwayes known.

Of theſe five parts any two being given, the reſt may be found.

In right angled Spherical Triangles there are XVI Caſes, 6 for finding the legs, 4 for the hypotenuſa, and 6 for the angles; but by projection may be reduced to the five Problems following.

PROBLEM 1.

The legs of a Right angled Spherical Triangle being given, to find the reſt.

TO avoid the multiplicity of Scheams, we will here make uſe of the 6 Figure in the ſecond Part of this Treatiſe, in which you have a right an oblique angled Spherical Triangle. The right angled Triangle is noted with the letters A B C, in which we will ſuppoſe the legs A B and A C to be given; and the oblique angles A C B and A B C with the hypotenuſa B C to be inquired.

Now that the ſolution of Spherical Triangles by projection may the better agree with their ſolution *Fig. 6.*

 upon

angular point propounded, will cut the primitive circle in I, make X W equal unto H I: a Ruler laid from F unto W, will cut the circle Z P X in α, and a Ruler laid from Æ unto α, will cut the primitive circle in a, make a b equal to the angle propounded, a Ruler laid from Æ unto b, will cut the Diameter H B in F, and *by the 3 Probl. of the 1 chapter*, having the two points Æ and F given, you may describe the circle R Æ F N, which is the onely difference between the projecting of this and the former angles, for there a Ruler laid from α unto R, did cut the Diameter H B in D the center of the primitive circle, and so the other circle to be projected became a straight line, as it ought, the angle being propounded to be conteined by two circles one oblique the other perpendicular to the sight; but both the circles were to be oblique in this variety, and therefore a Ruler laid from b unto Æ must not cut the Diameter H B in D: but some other point, as here at F by help whereof the circle R Æ F N must be described as hath been said. To project this angle is seldome propounded, but this angle being projected the measure thereof is often required; which may be thus effected.

Fig. 8. Having described the circle M a W by the directions of the 5 Problem, which must pass through a and n the poles of the circles Z E X and A E S, a Ruler laid from E to the intersections of that circle, with the other at f and g, will cut the primitive circle in h and l, the measure of the angle at E.

Or, a Ruler laid from E to a and n, will cut the primitive circle in m and q equal to h l the measure of the angle A E Z as before.

CHAP.

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Of theſe five parts any two being given, the reſt may be found.

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Now that the ſolution of Spherical Triangles by projection may the better agree with their ſolution *Fig. 6.*

 upon

upon the Globe; we will ſhew how thoſe arches of the Globe, by which the parts of this Triangle both given and required, were repreſented and meaſured upon the Globe it ſelf, may alſo be deſcribed and meaſured by projection: And upon the Globe it ſelf, the legs of this Triangle were repreſented one of them upon the braſs Meridian; from the interſection thereof with the Equinoctial at Æ, to the Zenith at Z, the other upon the Equinoctial from the interſection thereof with the Meridian at Æ to R, the hypotenuſa B C, is repreſented by the arch of the Quadrant of altitude Z R, let therefore the circle H Z B X repreſent the braſs Meridian, H D B the Horizon, A the North-pole, which ſuppoſe to be elevated above the Horizon at B, the quantity of the given leg A C, which being ſet from Æ upwards, will reach to Z the Zenith; the other leg A B muſt be numbred in the Equinoctial Æ æ from Æ to R, in this manner: ſet the quantity thereof by a line of chords, from Æ unto T, a Ruler laid from A unto T, will cut the Equinoctial Æ æ in the point R, ſo ſhall Æ R be the other leg given, and Z R X the three points by which to draw that circle whoſe center by the *6 propoſ. of the 3 chap. in the 1 Part* will be at C.

The Triangle Æ Z R being thus projected, the parts unknown Z R the hypotenuſe, and the oblique angles Æ Z R = A C B, and Æ R Z = A B C may thus be meaſured.

To find the Angle Æ Z R.

Lay a Ruler from Z to P, and it will cut the primitive circle in the point G, and the arch H G is the meaſure thereof.

To find the hypotenuſe Z R and the angle Æ R Z, make K K equal to H G, a ruler laid from Z to K, will cut the Horizon H B in the point F, the pole of the circle Z R X, and a ruler laid from F to R will

cut

cut the primitive circle in the points L and M the arch ZL = ZR, and the arch AM is the measure of the angle ÆRZ.

Or, The angle ÆRZ, may be otherwise measured in this manner: set off a Quadrant from L unto N, then will a ruler laid from F unto N, cut the Azimuth circle in the point E, and a ruler laid from R to E, will cut the limb in the point V, and the arch Xæ is the measure of the angle æRX = ZRÆ.

PROBLEM 2.

The Hypotenuse and one leg given, to find the rest.

IN the right angled Spherical Triangle ABC, let there be given, The hypotenuse BC and the leg AC:

To find the leg AB, and the oblique angles ACB and ABC.

Having drawn the primitive circle HZBX, and the two Diameters HDB and ZDX, set off the given hypotenuse BC from Z unto L, the Tangent of ZL being set from L to the Diameter XZ continued will give the point Y for the center of the circle LRW: then set off the given leg AC from Z unto Æ, and draw the Diameter ÆDæ, which will intersect the circle LRW in the point R, which is the third point by which to describe the circle ZRX. The Triangle ZRÆ being thus described; make BA equal to ZÆ, a ruler laid from A unto R, will cut the primitive circle in T, and ÆT is the measure of ÆR = AB.

A ruler laid from Z to P will cut the primitive circle in the point G, and the arch HG in the measure of the angle ÆZR = ACB. The angle ÆRZ = ABC may be measured, as hath been shewed in the last Problem.

PRO-

PROBLEM 3.

The hypotenusa and an angle given, to find the rest.

IN the right angled Spherical Triangle ABC, let there be given.

The hypotenuse BC and the angle ACB.

To find the legs AC and AB, and the angle ABC.

Fig. 6. Having drawn the primitive circle HZBX, and the two Diameters HDB and ÆDX set off the given hypotenuse BC from Z unto L, you may *by the 4 Prob. of the last chapter*, describe the circle LRW, then set off the given angle ACB from H unto G, a ruler laid from Z unto G, will cut the Diameter HDB in the point P, so may you describe the circle ZRX which will cut the circle LRW in the point R, through which point draw the Diameter ÆRD *æ*.

The Triangle ÆZR being thus described, The arch ÆZ will be the measure of the leg AC and ZA being made equal to HÆ, a ruler laid from A unto R, will cut the primitive circle in the point T, and the arch ÆT shall be the measure of ÆR equal to the leg AB, and the angle ÆRZ equal to ABC may be measured; as hath been shewed in the 1 Problem.

PROBLEM 4.

A leg and Angle given, to find the rest.

IN the right angled Spherical Triangle ABC, let there be given the leg AC the angle ACB.

To find the leg AB the angle ABC the hypotenusa BC.

Having drawn the primitive circle and the two Diameters as before; set off the given leg AC from Z unto Æ and draw the Diameter ÆD *æ*, the given angle ACB, set off from H unto G, a ruler laid from

Z

Z to G will cut the Diameter H D B in the point P, so may you describe the circle Z P X which will intersect the Diameter Æ D æ in the point R.

The Triangle Æ Z R being thus described, make Z A equal into Æ H, a ruler laid from A unto R shall cut the primitive circle in T, and the arch Æ T shall be the measure of Æ R equal to the leg A B.

To measure the hypotenuse Z R equal to B C, make X K equal to H G, a ruler laid from Z unto K will cut the Diameter H B in the point F the pole of the circle Z P X, and a ruler laid from F unto R will cut the primitive circle in L, and Z L shall be the measure of the hypotenuse Z R.

The angle Z R Æ may be measured, as hath been shewed in the 1 Problem.

But if the given leg be A B and the angle A C B opposite thereunto.

Set off the given angle from X to K and from K unto M, a ruler laid from Z unto K and M will find the pole of the circle Z P X at F, and the center at C, then set off the given leg A B from H unto O, a ruler laid from Z unto O will cut the horizon in I, with the extent I D cross the azimuth circle, this intersection will be at R, through which point and the center draw the Diameter Æ D æ. Then is Æ Z equal to A C, Z R and the angle Æ R Z may be measured, as hath been shewed.

PROBLEM 5.

The Oblique Angles being given, to find the rest.

IN the right angled Spherical Triangle A B C let there be given the obliques angles A C B and A B C.

To find the legs A B and A C and the hypotenuse B C.

¶ *This Problem upon the Globe we resolved by turning the angles into sides, and the sides into angles, and thus it may be resolved by projection also, as shall been shewed in the like Problem of oblique angled Spherical Triangles, we will here shew how to resolve it, without any such conversion.*

Fig. 7. Having drawn the primitive circle and the two Diameters as before, set off the angle A C B from H to G, a ruler laid from Z unto G shall cut the Diameter H D B in the point P, the center of the circle Z P X, may be found several wayes *by the 4 Probl. of the last Chap.* Make X K equal unto H G, a ruler laid from Z unto K will cut the Diameter H B in the point F the pole of circle Z P X, and all circles passing through the point F will cut the circle Z P X at right angles, and is therefore one point through which the other circle must pass; the Diameter of which other circle may thus be found: set the quantity of the angle A B C from Z unto V, a ruler laid from H unto V will cut the Diameter Z X in E, which is a third point by which to draw the circle H E B.

Or, Z V the measure of the angle A B C being set from B unto L and from L unto Q, lines drawn from H through Q and L shall cut the Diameter Z X continued in the pole, and center of the circle H E B; and if this circle be supposed to move upon the center untill it cut the point F or pole of the circle Z P X it will also cut that circle in the point Æ: and limit the Triangle Æ Z R, in which Z R in the limb is equal to the hypotenuse Z R in the azimuth circle of the former figure, and Æ Z in the azimuth circle of this figure is equal to Æ Z in the limb of the former, & Æ R in the circle *R Æ F* is equal to *Æ R* in the Diameter *Æ D æ* of the former figure.

The motion of this circle upon the *Jewel* is easie, and the projection thereof according to that supposed motion in this figure, is thus: upon the point D at the

extent

extent of the compasses from D to the center of the circle H E B, describe an arch in which the center S would alwayes be, if the line Z were moveable, then open your compasses to the extent of E, to the center of H E B, and setting one foot of that extent in F, move the other about until it will just touch the arch described, where upon such condition it resteth, make a mark, set now one foot of that extent in that mark and with the other describe the circle R F N, which because it passeth through the pole of the circle Z P K in the point F it doth also cut that circle at right angles in Æ and limit the triangle Æ R Z.

In which Z R is the hypotenuse measured in the limb; and a ruler laid from F unto Æ will cut the limb in I, and the arch Z I shall be the measure of Z Æ equal unto A C, and making D α equal to D Y, a ruler laid from α unto Æ shall cut the primitive circle in O, and the arch R O shall be the measure of R Æ equal to A B inquired.

CHAP. III.

Of the solution of the Oblique angled Spherical Triangles by the Stereographick projection.

IN Oblique angled Spherical Triangles there are 12 Cases, but in projection they may be reduced to the 6 Problems following.

PROBLEM I.

The three sides given, to find the Angles.

TO avoid the multiplicity of Schemes, we must there refer the Reader to the sixth Scheme of the

second part of this Treatise, in which the oblique angled Spherical Triangle is noted with the letters B C D, and in this Triangle we suppose the sides C B. C D and B D to be given, and the angles C D and B to be inquired.

Fig. 8. Having drawn the primitive circle H Z B X, and the two Diameters H D B and Z D X, as hath been already directed; Set off the side C D from Z unto A, and the side C B from Z unto L, and if you will from Z unto W, and (as hath been already shewed *Probl* 4. *chap.* 1.) draw the parallel L E W, the third side B D set from A unto G and K, and draw the parallel G E K where these intersect is the third point by which to draw the circles Z E X and A E S, as hath been taught in the 1 *Part.*

Otherwise (*by the* 5 *Probl.* 1 *chap.*) in this manner, draw the line D F, and make Z V = H F and X M equal unto Z V and draw the Diameter M D V, a ruler laid from V unto E will cut the primitive circle in N, make N O equal unto M N, a ruler laid from V unto O will cut the Radius D F being extended in P the center of the circle M Q V, a ruler laid from V unto Q will cut the primitive circle in R: make R T equal unto M R, a ruler laid from V unto T will cut the Diameter F D *æ* in Æ, make D *y* = D Æ, so is A *y* the radius of the circle A E S.

The circle M Q V cutteth D B in *a* the pole of the circle Z E X, a ruler laid from Z unto *a* will cut the primitive circle in *b*, and making *bc* = X *b*, a ruler laid from Z to *c* will cut D B being extended in the center of the circle Z E X, so have you described the Triangle A E Z the angles whereof may be measured as hath been shewed in the 7 *Probl. of the* 1 *chap.* of this Part.

PRO-

PROBLEM 2.

Two sides and their conteined angle being given, to find the rest.

IN the oblique angled Spherical Triangle B C D, the given sides are B C and C D the given angle B C D.

And the third side B D, with the angles C D B and D B C are inquired.

Having drawn the primitive circle H Z B X and the two Diameters H D B and Z D X, set off the given side C D from Z to A and from X to S, and draw the diameter A D S the complement of the given angle B C D set from H to *r*, a ruler laid from Z to *r* gives *t*, set off H *r* from X to *b*, and *b* to *c*, a ruler laid from Z to *b* and *c* will give *a* the pole and *d* the center of the circle Z *t* X, then set the side C B from Z to L, a ruler laid from *a* to L will cut Z *t* X in the point E, and then you may describe the circle A E S, as hath been shewed in the 5 Problem. chap. 1. or in the preceeding Problem of this chapter. Fig. 8.

The Triangle A E Z being thus described, we may find.

1 The angle Z A E, for a ruler laid from A to *u*, will give ♈, and *w* ♈ is the measure thereof.

2 The side A E, for if you make S ♉ equal to *w* ♈, a ruler laid from A to ♉ will give *n* the pole of A E S, and a ruler laid from *n* to E will cut the primitive circle in G, and the arch A G is the measure of A E.

3 The angle A E Z, for a ruler laid from E to *a* and *u* will cut the primitive circle in *m* and *q* and the arch *m q* is the measure of the angle A E Z.

PROBLEM 3.

Two Angles and a side between them being given, to find the rest.

IN the oblique angled Spherical Triangle B C D; let there be given the angles B C D and C D B and the side C D.

To find the angle C B D and the sides *B* C and *B* D.

Having drawn the primitive circle H Z *B* X and the two diameters H D *B* and Z D X, set off the side C D from Z unto A, and make the angle Z A E equal to the angle C D B, and the angle *A* Z E equal to the angle *B* C D, *by the 7 Probl. 1 chap. 4th. variety:* so may you draw the circles A *u* S and Z *t* X *by the 4 Problem 1 chap.* intersecting each other in the point E, and the Triangle *A E Z* being thus projected, the angle *A E Z* shall be equal to C *B* D, the sides *A E* and *E Z*, shall be equal, *D B* and *B C* and may be measured, as hath been shewed.

PROBLEM 4

Two sides and an angle opposite to one of them being given, to find the rest.

IN the oblique angled Spherical Triangle *B* C D, let there be given, the sides C D and C *B* with the angle C B D.

To find the side *B* D and the angles *B* C D and *B* D C.

Fig. 9. Having drawn the primitive circle *H Z B X* and the two diameters *Z D X* and *H D B*, set off the side *B C* from *Z* unto *A*, and make the angle *A* Z *E* equal to the angle *C B D by the 7 Problem 1 chapter*, and the side *C D* set from *Z* to *M* or *N*, and draw the parallel *N E M by the 4 Probl. 1 chap.* which will intersect the circle

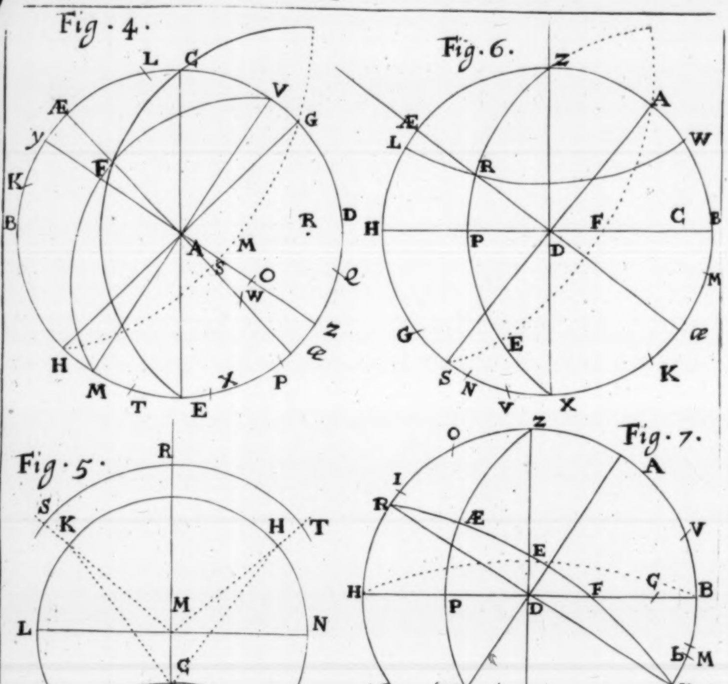


Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
pag: 191

circle Z *t* X in E, so may you also draw the circle A E S and compleat the triangle A E Z, in which the side E Z shall be equal to B D, the angle Z A E shall be equal to B C D and A E Z equal to B D C and may be measured, as hath been shewed. Yet in the solution of this Problem, it will be necessary, that the affection of the angle opposite to the other side be known, or otherwise there will arise a double solution, and both true.

PROBLEM 5.

Two angles and a side opposite to one of them being given, to find the rest.

IN the oblique angled Spherical Triangle B C D, let there be given the angles B C D and C D B with the side C B.

To find the sides C D and D B with the angle C B D. *Fig.* 1

Having drawn the primitive circle *H Z B X*, and the two diameters *Z D X* and *H D B*, make the angle *A Z E* equal to *B C D* and the side *Z E* equal to *B C*, as hath been shewed, and the angle *A B C* equal to *C D B*, and describe the circle *H C B* whose Semidiameter is *C F*. At the distance of *D F* describe the circle *L F G K*, in which the center of the circle *H C B* must be supposed to move, open your compasses to the extent of *C F*, and setting one foot of that extent in *E*, move the other about, until it cut the arch *L F G K*, which will be in the two points *F* and *G*, and therefore the center of the circle, which constitutes the third side of the triangle must be in one of these: in *F* if the side opposite to the other angle given be more than a Quadrant, in *G* if less. *Fig.* 9.

The triangle *A Z E* being thus compleated, the side *A Z* shall be equal to *C D*, the side *A E* to *D B*, and the angle *A E Z* to *C B D*, and may be measured as hath been shewed.

PRO-

PROBLEM 6.

The three angles being given, to find the sides.

IN the Oblique angled Spherical triangle *B C D*, let there be given the angles *B. C.* and *D*, and let the sides *B C. C D.* and *B D* be inquired.

To resolve this Problem, the angles must first be turned into sides, as hath been shewed in the 10 *of the* 12 *chap.* 2 *Part*, and then the sides may be projected, and the angles inquired found, as hath been shewed in the 1 *Problem of this chapter.*

And thus we have breifly run over the Doctrine of Spherical triangles by the Stereographick or circular projection, as for the variety of Questions which each triangle will afford, and the position of those triangles upon the Globe, from whence the most useful Problems whether *Astronomical* or *Geographical* are deduced, the Reader may be in some measure satisfied by what hath been said in the Use of the Globes; and need not be again repeated here.

But instead thereof we will here adde two other uses of the Globe, which were omitted there, and purposely reserved for this place, because they may be better explain'd by projection then many words.

The first is, how to erect a Figure of the twelve Houses of Heaven, according to *Regiomontanus*, or that which is called the *Rational way*, and now used by the the *Astrologers* of this age.

The other is how to describe the several sorts of Dials that any flat or plain is capable of.

CHAP. IV.

To erect a Figure of the twelve Houses of Heaven.

AMong the ſeveral wayes by which a Figure of Heaven may be erected, that hath now the beſt acceptance, which divideth the Equinoctial into twelve parts by circles meeting at the interſections of the Meridian and Horizon, and this way is by *Regiomontanus* called *Modus Rationalis*, I ſhall not defend the Rationality of it, but ſince it is ſo well liked, I ſhall here ſhew how it may be done upon the Globe it ſelf and by projection.

That a Figure of Heaven may be erected upon the Globe it ſelf, there muſt be a Semicircle, ſo faſtned to the Horizon at the interſections thereof with the braſs Meridian, as that it may be moved either higher or lower, and that upon either ſide of the Globe at pleaſure; upon a Globe thus furniſhed, a Figure of Heaven may be erected by the following directions.

To find the *Cuſpides* of the twelve Cœleſtial Houſes, the latitude of the place, the hour of the day, the true place and Right aſcenſion of the Sun, for the time propoſed, are ſuppoſed to be given: Let therefore the latitude of the place be 51 $\frac{1}{2}$ and the given time, the 20th. of *March* one hour after noon 1660 the place of the Sun at that time being in *Aries* 10 degrees, and 32 hundred parts: and therefore his Right aſcenſion deg. 9.47, *by the* 10 *Problem in the Ecliptical Triangle, chap.* 14 *in the uſe of the Globe*, unto which if you adde 15 deg. more, for the hour given, the aggregate deg. 24.47 is the Right aſcenſion of the *Medium Cœli*, or tenth houſe, and by adding of 30 deg. to the Right

Ascension of the tenth house, the oblique ascension of the eleventh house will be deg. 54.47, and by the continual addition of 30 deg. the oblique Ascension of the 12 House will be 84.47 of the 1 House 114.47, of the 2 House 144.47, of the 3 House 174.47.

Now then to erect a Figure of Heaven upon the Globe, elevate the Pole of the World deg. 51 ½ above the Horizon, and the Sun being in ♈ deg. 10.32, bring deg. 10.32 of ♈ to the brass Meridian, and the Judex of the hour-circle to 12. And because it is in the afternoon, turn the Globe West-ward, tell the Index-point unto one hour after noon, then stay the Globe in that position, and observe what degree of the Ecliptick doth cut the Meridian (which in this Example will be of ♈ degr. 26.40) and that is the *Cuspe* of the 10 House: and the degree of the Ecliptick, viz. 14.11 ♌, which cutteth, the Horizon is the *Cuspe* of the Ascendant; And lifting up the Semicircle before described, till it pass over 30 deg. of the Equater from the Horizon upwards, it will cut deg. 19.32 of ♋, for the *Cuspe* of the 12 House: and lifting up that Semicircle again till it pass over 30 deg more of the Equinoctial, it will cut degr. 10.91 of ♊ for the *Cuspe* of the 11 House. And turning this Semicircle of position to the West-side of the Meridian, if you let it fall towards the Horizon, till it pass over 30 degrees of the Equator from the Meridian, it will cut deg. 25.37 of ♓, for the *Cuspe* of the 9 House; and letting the circle of position fall yet lower, till it pass over 30 degrees more of the Equator; it will cut deg. 3.81 of ♓ for the *Cuspe* of the 8 House, the other six Houses, are the same degrees and parts of the opposite signes; as shall be further explain'd, in erecting the same Figure by projection.

Fig. 10. Having drawn the primitive circle H Z R N representing the Meridian, and the two Diameters H O R, and Z O N, set off the height of the pole deg. 51 ½, from R unto P, and from N to S, and draw the Diame-

ters

ters P O S for the Axis of the World, and Æ O α for the Equator: This done, the Right Aſcenſion of the Mid-heaven being given as before deg. 24.47, by the Ecliptical triangle, *the 6 Probl* you may find the *Cuſpe* of the 10 Houſe to be in deg. 26.40 of ♈ as before, and the Declination of deg 26.40 of ♈, *by the 17 Probl.* of the ſame triangle, to be deg. 10.25, ſet therefore deg. 10.25 from Æ unto B, and from α to C, and draw the Diameter B O C and D O E at right angles thereunto.

And becauſe the fourth Houſe is directly oppoſite to the tenth, that is, in deg. 26.40 of ♎, ſet deg. 26.40 from C to F, a ruler laid from E to F will cut the Diameter B C in G, and then you may draw F G H, as hath been directed, cutting the Equator Æ O α in the point H, and ſo you have the three points B H C, by which to deſcribe the arch of the Eclipticк B K H C.

And becauſe the circles of poſition muſt cut the Equator at 30, and 30 degrees above the Horizon, ſet 30 degrees from Æ to L, and from L to M, a ruler laid from P to L and M ſhall cut the Equator at N and Q, and then you may deſcribe the circles of poſition H N R and H Q R, make O S equal O Q and O T equal O N, and ſo you may deſcribe the circles H T R and H S R, and where theſe circles doe cut the arch of the Ecliptick B K C there are the *Cuſpides* of the Cœleſtial Houſes.

Thus a ruler laid from V the Pole of the Ecliptick, to the interſections at *X. Y. W. f. l.* will cut the primitive circle in *a. b. c. d.* and *e.* and the arches B *a* = B *X*. B *b* = B *y*. B *c* = B *w*. B *d* = B *f*. and B *e* = B *l* being added to ♈ B will give you the *Cuſpes* of the 11. 12. 1. 2. and 3d. Houſes, the other ſix are the ſame degrees and parts of the oppoſite Houſes: as in the Table following.

Six Oriental Houses.					*Six Occidental Houses*			
10	*Deg.*	26.40	♈	*In the opposite points, are*	4	*Deg.*	26.40	♎
11		10.91	♊		5		10.91	♐
12		19.32	♋		6		19.32	♑
As		24.11	♌		7		14.11	♒
2		3.81	♍		8		3.81	♓
3		25.27	♍		9		25.27	♓

In Arithmetical Calculation the heigth of the Pole above each circle of position is required, the which in this projection, is easily found; as if it were required to find the height of the Pole above the circle of position H Q R, the pole of that circle is *r*, and so you have the 3 points S *r* P to describe that circle by, which will cut the circle H Q R at right angles in the point *t*, and the arch P *t* is the height of the pole above that circle, and being measured according to the directions already given, will be 47.46 degrees, for the twelfth and second Houses: in like manner the height of the pole above the circles of position, for the 11 and 3 Houses will be found to be 32.18 deg. As for the Arithmetical work, see the *16 chap. of the first Part of my Astronomia Britannica*, and thou wilt find satisfaction, not only in this, but several wayes for perfecting the other work also.

CHAP. V.

Of the several sorts of Dials.

SUn Dials are either such as shew the hour of the day, by the suns altitude, or by his shadow, the hour of the day by the Suns altitude, is shewed by Quadrants, Rings, Cylinders, and such like, of which we shall make no farther mention here.

In

It is our present purpose to speak onely of such Dials as shew the hour by the shadow of a Gnomon, or Style parallel to the Axis of the World. And these are all of them projections of the Sphere upon a plain which lieth parallel to some Horizon or other. In a Dial therefore that is projected upon any plane, however situate, the center thereof doth represent the center of the earth, and the Gnomon which casteth the shade, representeth the Axis of the World, and ought to point directly to the two poles.

The plains upon which these Dials may be projected which shew the hour by the shadow of a Gnomon or Style, are of three sorts.

1 Parallel to the Horizon, as is the Horizontal onely.

2 Perpendicular to the Horizon, as are all erect plains, whether they be such as are direct *North* or *South*, *East* or *West*, or such as doe decline from those points of *East*, *West*, *North* and *South*.

3 Inclining to the Horizon, or rather reclining from the Zenith, and these are direct plains reclining and inclining *North* and *South*, and reclining and inclining *East* and *West*, or declining, reclining and inclining plains.

How the hour-lines may be projected upon all these several plains, shall be shewed in order, and first upon that which is parallel to the Horizon, and therefore called the Horizontal plain: In which as to the Dial, it will be sufficient to project the Meridians onely, but because we intend this as the foundation of the rest, we will shew how all the several plains are represented upon the Globe, as it may be projected upon the plain of the Horizon.

CHAP. VI.

To project the Meridian or hour-lines upon the Horizontal plain, with the other lines usual in that projection.

SUppose the Globe being elevated to the height of the pole, to be pressed flat down into the plain of the Horizon, then will the outward circle or limb N E S W represent that Horizon, and all the circles conteined in the upper Hemisphere of the Globe may be artificially contrived and represented thereon, as the Meridians, Azimuths, Almicanters, Parallels, Equator, Ecliptick, Tropick, Circles of position and such like, the whichin this projection are thus distinguished.

Let *Z* be the Zenith of the place, and center of the fundamental and horizontal circle N E Z W, let N Z S be the Meridian, P the pole of the World, elevated above the *North* part of the horizon N, here at *London* 51.53 the complement whereof is P *Z* 38.47 the distance between the pole and the zenith, E *Z* W the prime vertical, or a circle passing from the *East* through the zenith to the *West* point of the horizon. D Z G and C *Z* V some other intermediate azimuths, N O S a circle of position, E K W the Equator, the distance whereof from *Z* is equal to P N the height of the pole, or from S equal to P *Z* the complement thereof, M B Q X the Tropick or parallel of *Cancer*, A F *H* the Tropick of *Capricorn*, the rest of the circles intersecting each other in the point P are the meridians or hour-circles cutting the horizon and other circles of this Diagram so in the Scheme, as they do in the Globe it self.

Amongst

Amongst these, the Azimuths onely in this projection become straight lines, all the rest remain circles, and are greater or lesser according to their natural situation in the Globe. By the straight lines are represented all erect plains, whether Direct or Declining: by the great circles all the rest, both jacent, reclining, and declining reclining which for more plainess sake, may be thus particularly described.

E Z W the prime vertical, or azimuth of *East* and *West*, representeth all *South* and *North* plains, which are perpendicular to the horizon, and cross the meridian in the zenith at right angles; N Z S the meridian or azimuth of *South* and *North* represeuteth all *East* and *West* planes which are perpendicular to the horizon, as the former, and cutteth the prime vertical in the zenith at right angles: D Z G an azimuth lying between these cardinal points, representeth any declining plane, which is also perpendicular to the horizon, but cutteth the meridian in the zenith at oblique angles, from whence the poles and axis of the plane C Z V deviateth as much as the plane it self D Z G declineth from the prime vertical, and these be all the varieties of erect planes.

There are further more three sorts of reclining and inclining planes, and they are either *North* or *South* reclining, or *East* and *West* reclining, or declining reclining planes.

The first sort, is represented by the equator E K V V, which cutteth the meridian at right angles, but reclineth from the zenith deg 51.53 equal to the latitude of the place, and lyeth open to the *North*, and the poles thereof in the *North* part of the meridian, therefore called a *North* reclining 51.53 from Z to K, if you suppose this circle to be turned over, it will fall between N and Z and represent a *South* reclining as much.

The second sort is represented by the circle of position or prickt circle N O S, which cutteth the prime verti-

vertical at right angles, but reclineth from the zenith 30 degrees, and lyeth open to the *East*, and the poles thereof in the *East* part of the prime vertical, therefore called an *East* reclining 40 degrees from Z to O: if you suppose this circle to be turned over, it will fall between Z and E, and represent a *West* reclining as much.

The third sort is represented by the pricked circle D L G which cutting C Z V the azimuth and axis of the plane D Z G at right angles in L is oblique to all the rest of the circles, but reclineth from the zenith 35 deg. and lyeth open to the *North*, as the former, and the poles thereof in the Northern part of the Heavens, therefore called a *North* reclining 35 deg. from Z to L upon the Azimuth passing by the poles of the plane C V declining 30 degr. from N and S to C & V; if you suppose this circle to be turned over, it will fall between G and Z and represent a *South* reclining declining as much.

The inclining planes of all sorts are but the opposite sides of the reclining, being the same counted from the zenith and nadir or their complements, reckoned from the Horizon, and are represented by the very same circles, the reclination and inclination of both being arches of the same azimuth, passing by the poles of the plane, comprehended betwixt the zenith or nadir and the plane, as is Z K for the *North* direct, and Z L for the *North* declining, so that whatsoever shall be said of the one, may also be understood of the other respectively.

The jacent plane or Horizontal chiefly intended in this Diagram is represented by the limb, or outward circle of the Scheme N E S VV.

Lastly, the circles crossing each other in the point P, and continued to the Horizon, are the Meridians or hour-circles, issuing from the *North* pole, and properly intersecting the *North* sides of the planes E Z VV and D Z G, whensoever therefore you deal with *South*

planes

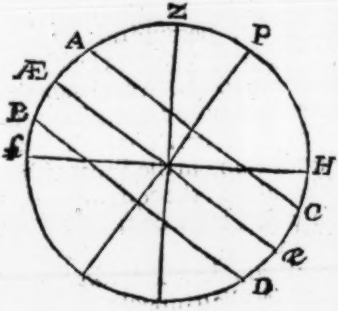
planes you may turn the Scheme about, and suppose P to be the *South* pole, and E Z W to be a *South* plane, which before was a *North*, or else invert the order of the sides and hours, taking the *East* side for the *West*, &c. and it will serve the turn as it stands.

The making of this Scheme is easie, open your Compasses to the extent of 60 degrees in your line of chords, and with that Semidiameter draw the circle N E S W; cross it at right angles in Z with the lines N Z S and E Z W; that done, because this circle representeth the Horizon, the center thereof the Zenith, the diameter N Z S the Meridian, before we can proceed any further, we are to know at what distance the parallels are to be set from the center. It is meet therefore first to enquire the same thing in the Sphere (*viz.*) how many degrees each parallel of the equator doth lie from the Zenith of the place, both on the *South* and *North* parts of the Meridian.

In this Scheme the work will be plain, let the circle *h* Z H D represent the Meridian suppose of *London*, Z the Zenith P the pole, *h* H the Horizon, Æ *æ* the Equator, A C the Tropick of *Cancer*, B D the Tropick of *Capricorn*; each declining 23 ½ degr. from the Equator Æ *æ*, the Question now is, how far the pole at P, the Equator at Æ *æ*, and these two parallels are from the zenith point Z both wayes, towards A B the *South*, and towards C D the *North*? for answer, we must consider, that from Z to Æ Southwards is the latitude of the place (as of *London* if you will 51 ½ deg.) and from Z to *æ* Northward is the complement of the latitude to a Semicircle, *viz.* 128 ½ d. & these are the *South* and *North* distances of the Equator from the zenith, or beginings of *Aries* and *Libra*. Now therefore to draw the circle E K W representing the Equator, take from your scale the tangent of 25.75 deg. the half of 51.50 degr. and set them from Z to K Southward, and the tangent of 64.25 deg. the half of 128.50 deg. and set them from Z to R Northwards, in T the

middle between these two shall be the center of the circle E K W representing the Equator; or thus, having found the point K by the tangent of 25.75 deg. the tangent of the co-latitude 38.50 deg. being set from Z Northward, or the Secant of the latitude of 38.50 deg. being set from K Northward shall give the center of that circle at T, as before.

And to describe the parallels A C and B D we must consider that A Æ and Æ B, and so C æ and æ D doe upon our supposition decline 23 $\frac{1}{2}$ deg. from the Equator. So then if we take 23 $\frac{1}{2}$ out of Æ Z the latitude of the place 51 $\frac{1}{2}$, there will remain A Z 28 deg. and so much is the parallel of *Cancer* distant from the zenith at *London* on the *South* part of the Meridian.



Again, because Z æ is 128 $\frac{1}{2}$ deg. and æ C 23 $\frac{1}{2}$ degr. taking æ C out of æ Z, there will remain 105 deg. and so much is the same parallel of *Cancer*, distant from the zenith on the *North* part, and hence the Tropick of *Cancer* may be thus described. The tangent of half 28 deg. that is, of 14 set from Z unto *y*, giveth the point *y* in the Meridian, and the tangent of half 105 d. that is, 52 $\frac{1}{2}$ set from Z unto *b* will give the point *b* on the *North* part of the Meridian and *b y* is the diameter of the circle, and *a y* the half hereof is the radius.

To describe the *South* parallel B D, belonging to *Capricorn*, add Z Æ 51 $\frac{1}{2}$ deg. to Æ B 23 $\frac{1}{2}$ deg. the sum is 75 degrees, shewing the distance of *Capricorn* from the zenith on the *South* part of the Meridian, to be 75 deg. and Z æ 128 $\frac{1}{2}$ deg. add to æ D 23 $\frac{1}{2}$ deg. gives the

the distance of the same parallel of *Capricorn* from the zenith to be 152 deg and the tangent of half 75, that is, of 37 $\frac{1}{2}$ d. set from Z unto F giveth the point F on the *South* part of the Meridian, and the tangent of half 152, that is, 76 deg. set from Z upwards will give the other point of the diameter the half whereof F *c* is the Radius.

The straight lines C Z V or D Z G are put upon the limb, by help of the chord 30 deg. distant from the cardinal points N E S W and must cross each other at right angles in Z, representing two azimuths equidistant from the Meridian and prime vertical.

The center of the circle of position N O S reclining 40 deg. from the zenith, may thus be found, the tangent of half the reclination, that is, of 20 deg. will give the point O: and the secant of the complement of reclination, that is, of 50 d. will reach from O unto *d* the center of that circle.

In like manner, the center of the circle D L G, reclining from Z unto L 35 deg. may thus be found, the tangent of half 55 degr. that is, 17 $\frac{1}{2}$ degr. set from Z unto L, will give the point L in the Azimuth circle C Z V, and the secant of the complement of 35 degr. that is, of 55 deg. set from L towards C, will give the center of that circle.

Lastly, the Meridians or hour-circles are thus to be drawn, the distance between Z the zenith, and P the pole of the World is 38 $\frac{1}{2}$ deg. and the tangent of the half thereof, that is, of 19 $\frac{1}{4}$ deg. set from Z Northward, will give the point P in the Meridian, and the secant of the latitude 51 $\frac{1}{2}$ deg. set from P Southward will give *m* the center of the 6 of the clock hour circle W P E, and a line drawn at right angles to the Meridian N S on the point *m*, shall be a tangent line to the Radius P *m*, and the line in which the centers of the other hour-circles will fall, that is, the tangent of 15. 30. 45. 60 and 75 degrees, being set upon that line from the point *m* both wayes, that is, 15 from *m* to 5

 and

and 7, and 30 from *m* to 4 and 8, will give the true centers of those hour-circles; thus 5 upon the line 8 *m* 4 is the center of the hour-circle 5 P 5, and 7 is the center of the hour-circle 7 P 7, and so of the rest.

But because this way doth presuppose a tangent line fitted to the Radius, the centers of these hour-circles may be otherwise found in this manner, make P *m*, or any other extent the Radius, and upon the point P describe the circle *n m q*, and divide the Quadrants *m q*, and *m n* into 6 equal parts, a ruler laid from P to those divisions, shall cut the line 8 *m* 4 in the centers of the hour-circles, as before.

Or, the intersections of the Meridians with the Horizon may thus be found: Having described the Equator R E K W, as hath been shewed, divide the primitive (which in this case is the Horizontal) circle into 24 equal parts, a ruler laid from Z the center of the Scheme to those equal divisions will divide the Equator into 24 unequal parts; and a ruler laid from P the pole of the world to those unequal parts in the Equinoctial, will cut the primitive circle into 24 unequal divisions also, the lines drawn from Z through those divisions, shall be the hour-lines of that Dial, to which there is nothing wanting but the Cocke or Style, whose height above the plane will be equal to the latitude of the place, and stand directly over the line Z S or noon line.

The projection of the *North* and *South* Dials direct is the same with the Horizontal, save onely that in these the distance from Z the Zenith, to P the pole of the world must be equal to the co-latitude, whereas in the Horizontal it was equal to the height of the pole; what hath been therefore said of the horizontal, is to be understood of these also.

Fig. 11.

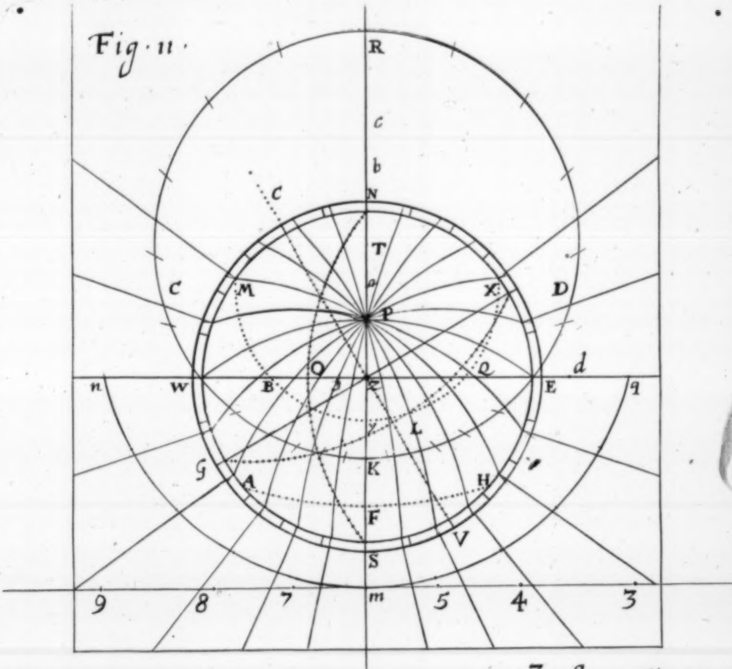


Fig. 9.

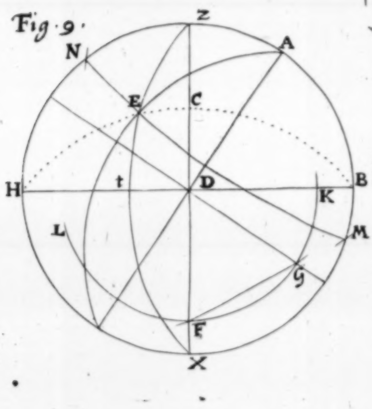
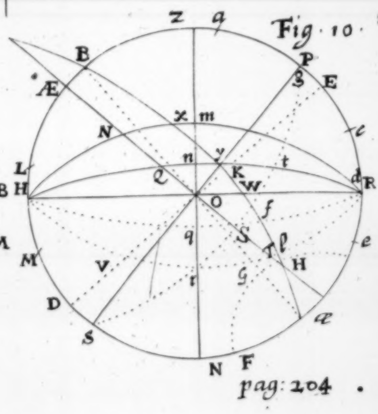


Fig. 10.



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CHAP. VII.

To project the Hour lines upon the East *and* West *planes direct.*

THese Dials doe lie in the plane of the Meridian, and the hour-lines are parallel to the Axis of the world, and have therefore no centers, but may be described in this manner. Having drawn the primitive circle ZESVV representing the meridian, and the two Diameters ZAS, and VVAE, make EP equal to the latitude of the place suppose 51 $\frac{1}{2}$ and draw the Axis PAB and CAV at right angles thereunto: this done, divide the primitive circle into 12 equal parts beginning at P or B, a ruler laid from P or B to those divisions, shall cut the Equator in the points D. E. V. G. H. K. and C. and right lines drawn through those points parallel to the Axis PAB shall be the hour-lines of this Dial, the Axis it self being the hour of 6, and the substiler line. If it be an *East* plain, you may have all the morning houres, as here you see, if a *West*, this plane will hold the hours from noon to Sun-set, the height of the Stile above this plane is alwayes equal to the Radius of your circle, and must stand directly over the line PB, or Axis of the world. *Fig. 12.*

CHAP. VIII.

To project the hour lines upon a South *or* North *plain erect, declining* East *or* West.

EVery erect plain lieth under some Azimuth or other, and those onely are said to Decline which differ from the Meridian and prime vertical, this plane

in the Horizontal projection, is represented by the right line G Z D the poles whereof are C and V, the declination thereof from the *South* Easterly let be D E or *North* westerly W G 45 degrees. Now the hour lines of this Dial may be thus projected upon the plane it self.

Fig. 13. Having drawn the primitive circle Z E N W, and the two Diameters Z A N and VV A E, set off the declination given 45 degrees from W unto B, a ruler laid from Z unto B shall cut the Horizon in C, set W B from N unto D, and from D unto E, a ruler laied from Z to D, and E shall cut the Horizon W A E in G the pole and E the center of the Meridian circle Z C N, the distance of the pole of the World from the zenith, 38 $\frac{1}{2}$ deg. set from Z unto K, a ruler laid from G to K shall cut the Meridian in P the pole of the World, and aline drawn from P through the center at A, shall be the substile; from E the center of the meridian Z C N, draw the line E L at right angles, to the substiler, which shall be the tangent line of your plane in which the center of all the hour-circles must fall: And are easily found in this manner, upon P as a center, describe a circle at what extent of the compasses you think best, suppose P L, this circle must be divided into 24 equal parts, beginning your division where a ruler laid from P the pole of the World to E the center of the Meridian shall intersect this new described circle, as here in the point M, here therefore I begin to divide the circle, and the distance from M to Q is the first division, and a ruler laid from P to Q will cut the tangent line in the point I which is the center of the first hour from noon *viz.* 11, or 1 a clock, E being the center of 12 a clock (and here the vertical circle) and so of the rest: your Scheme being thus furnished with the Meridians or hour-circles, a ruler laid from the center A to these several intersections with the primitive circle, or circle of the plan and straight lines extended thereby will be the true hour-lines required. Bute

But because the centers of some of the meridians or hour-circles, will many times fall very remote, and beyond the extent of an ordinary sheet of paper, we will here shew another way for the describing of them by which that inconvenience will be at least in part prevented.

By what hath been already said, the pole of the meridian at G is already found, which is the *West* point of the horizon, in a *North* plane declining *East*, and the *East* point in a *North* plane declining *West*; and therefore to describe the Equinoctial cannot be difficult, for considering, that the Equinoctial in the Globe doth alwayes pass through the *East* or *West* points of the Horizon, and cutteth the meridian at right angles in all Latitudes, and that the pole of the meridian is alwayes in these Dials either the *East* or *West* point of the horizon the Diameter and Semidiameter of that circle is easily found, *by the 4 Problem of the* 1 *hereof*: by making ZH = ND, and drawing a line from Z through H till it cut the horizon W A E continued, the distance from G to that intersection being bisected, and a perpendicular raised upon the point of bisection to the horizon shall intersect the Axis in the center of the Equinoctial, or the *East* and *West* being thus found, if you make WR = ZK, a ruler laid from G unto R, will cut the meridian in S, and so you have the three points, to wit, the *East* and *West* points and the point S, by which to describe the Equinoctial, and a ruler laid from P the pole of the World to S, the intersection of the Equinoctial with the meridian will cut the primitive circle in that point where you must begin to divide it into 24 equal parts, and a ruler laid from A the center to these equal divisions will cut the Equinoctial into 24 unequal parts, a ruler laid from P the pole, to those unequal divisions of the Equinoctial will cut the plane or primitive circle into 24 unequal parts, or points in which the meridians or hour-circles will cut the plane, by which points they may

be:

be drawn without finding the centers, by help of a ſteel-bow, the form and uſe whereof is ſufficiently known.

The hour-lines being thus deſcribed P V is the height of the ſtile, which being meaſured according to Art, muſt ſtand over the ſubſtile P L, making an angle therewith at P equal to the height thereof, and this Dial is alſo finiſhed.

CHAP. IX.

To project the hour-lines upon any direct plane reclining or inclining Eaſt *or* Weſt.

Hitherto we have ſpoken of ſuch planes, as are either parallel or perpendicular to the Horizon, all which except the horizontal, doe lie in the plane of ſome Azimuth or other. The reſt that follow are reclining or inclining planes, according to the reſpect of the upper or neerer faces of the planes, in thoſe that recline, the baſe is a line in the plane parallel to the Horizon or Meridian, and alwayes ſituate in ſome Azimuth or other: thus the baſe of the *Eaſt* and *Weſt* reclining planes lie in the meridian or *South* and *North* Azimuth, and the poles thereof in the prime vertical, but the plane it ſelf in ſome circle of poſition (as it is Aſtrologically taken) which is a great circle of the Sphere, paſſing by the *North* or *South* interſections of the meridian and horizon, and falling *Eaſt* or *Weſt* from the Zenith upon the prime vertical as much as the poles of the plane are elevated, and depreſſed above and under the horizon; This is repreſented in the horizontal projection by

by the circle NOS, and the hour-lines upon this plane may be thus projected.

Having drawn the fundamental circle NESW representing a *West* reclining plane 35 degr. and the two Diameters NAS and WAE, set off the reclination from N to B, from W to D, and from D to L, a ruler laid from W to B shall cut the line NS in Z the Zenith, and a ruler laid from F to D and L shall cut the same line NS in F the pole, and C the center of the meridian circle WZE, and the co-secant of the reclination set from F upwards, will give the center of the horizon WFE. *Fig* 14.

To find where the pole of the world shall fall, set the elevation thereof from W to H $51\frac{1}{2}$, a ruler laid from F the pole of the meridian unto H, will cut the meridian in P the pole of the world, and the line PA shall be the substile.

From C the ceuter of the Meridian draw the line CK perpendicular to the substile PF, and at any distance suppose of PK describe a circle upon the point P, which must be divided into 24 equal parts, (as was said in decliners) and a ruler laid from P the pole of the world to C the center of the Meridian will cut that circle in M, at which point you must begin to divide, a ruler laid from P to those divisions will cut the tangent line CK being extended in the centers of the meridians or hourcircles, which may be otherwise drawn by help of the Equinoctial, as hath been shewed in Decliners; as to the perfecting of this Dial there is nothing wanting but the height of the pole above the plane, represented by PQ, and being measured as hath been shewed, is the height of the stile, which must be erected upon the substilar PA, and the Dial is finished.

CHAP. X.

To project the hour-lines upon any direct South *reclining or inclining plane.*

AS the base of *East* and *West* reclining or inclining planes doe alwayes lie in the meridian of the place, or parallel thereunto, and the poles in the prime vertical; so doth the base of *South* and *North* reclining or inclining planes lie in the prime vertical or Azimuth of *East* and *West*, and their poles consequently in the meridian. Now if you suppose the circle of position (which Astrologically taken is fixed in the intersection of the meridian and Horizon) to move about upon the Horizon, till it comes into the plane of the prime vertical, and being fixed in the intersection thereof with the horizon, to be let fall either way from the Zenith upon the meridian it shall truly represent all the *South* and *North* reclining and inclining planes, of which there are six varieties, three of *South*, and three of *North* reclining; for either the *South* plane doth recline just to the pole, and then it becomes an Equinoctial, because the poles of this plane doe then lie in the Equinoctial, some call it a polar plane: or else it reclineth more and less than the pole, and consequently the poles of the plane are above and under the Equinoctial, somewhat differing from the former.

In like manner, the *North* plane reclineth either just to the Equinoctial, and then it becometh a polar plane, because the poles of that plane doe lie in the poles of the World; some term it an Equinoctial plane. Or else it reclineth more or less than the Equinoctial, and consequently the poles of the plane are above and under.

under the poles of the world, ſomewhat differiug from the former.

The firſt of theſe varieties, which I call an Equinoctial plane, is in the horizontal projection, repreſented by the ſix of a clock hour-circle E P W, wherein you may obſerve out of the Scheme it ſelf, that none of the other hour-circles doe cut the ſame, and therefore you may conclude (as in the 7 Chap.) that the hour-lines thereof have no center to meet in, but muſt be parallel to one another, as hath been ſhewed in the *Eaſt* and *Weſt* Dials.

And becauſe this Dial is no other but the horizontal of a right Sphere, where the Equinoctial is zenith, and the poles of the world in the Horizon, therefore it is not capable of the ſix a clock hour (no more than the *Eaſt* and *Weſt* are of the 12 clock hour) which vaniſh upon the planes, unto which they are parallel: and the 12 a clock hour is the middle line of this Dial (becauſe the meridian cutteth the plane of ſix a clock at right angles) which the Sun atteineth not till he be perpendicular to the plane. This Dial is to be made in all reſpects as the *Eaſt* and *Weſt* are, onely changing the number of the hours: for ſeeing the 6 a clock hour in which this plane lieth, croſſeth the 12 a clock hour at right angles, in which the *Eaſt* and *Weſt* plane lieth, the reſt of the hour-lines will have equal reſpect unto them both; ſo that the fifth hour from 6 of the clock is equal to the fifth hour from 12, the fourth to the fourth, and ſo of the reſt; and therefore the hour-lines may be projected upon this plane by thoſe directions already given, in the *Eaſt* and *Weſt* Dials.

The ſecond variety, is of a *South* plane reclining from the zenith leſs than the pole, this might have been repreſented in the horizontal projection by a circle drawn like the ſix a clock hour, but between the pole of the world and Z the zenith of the place, upon the plane it ſelf the hour-~~lines~~ may be thus projected.

Having drawn the primitive circle N E S W,

Fig. 15. representing a *South* reclining plane 25 degr. and the two Diameters N A S and E A W; subtract the reclination given 25 degrees from the distance of the pole of the world from the zenith, which in our latitude of *London* is 38 $\frac{1}{2}$, and their difference is 13 $\frac{1}{2}$ which set from S unto B, from W unto C, and from C unto D, a ruler laid from E unto B, will intersect the meridian N S in P the place of the *South* pole, and a ruler laid from E unto C and D will give the pole of the *6* a clock hour-circle at F, and the center at G, the centers of the other hour-circles will fall in the Tangent C H drawn at right angles to the point G, and center of the *6* a clock hour-circle, as hath been shewed in the horizontal, S P is the poles elevation or height of the stile, according to which the cocke of the Dial must be placed over the substiler-line N A S, and the Dial is finished.

The third variety, is of a *South* plane reclining from the Zenith more than the pole, this might have been represented in the horizontal projection, by a circle drawn like the six a clock-houre, but between N the *North* part of the meridian, & P the pole of the world, upon the plane it self, the projection of the hour-circles is so like the former, that more need not to be said of it, onely note, that in these the *North* pole will be elevated, the *South* in the other, and therefore to find the place of P the pole of the world, you must subtract the distance of the pole from the Zenith, which suppose to be 38 $\frac{1}{2}$ as before, from the reclination given which let be 55 degr. the remainer is 16 $\frac{1}{2}$ the distance of the *North* pole from the intersection of the Meridian with the plane or primitive circle: the rest of the work is the same with the former, and therefore needeth no further direction.

CHAP.

CHAP. XI.

To project the hour-lines upon any direct North *reclining or inclining plane.*

THe direct *North* reclining planes have the ſame variety that the *South* had; for either the plane may recline from the Zenith juſt to the Equinoctial, and then is a polar plane, as I called it before, becauſe the poles of the plane lei in the poles of the world; or elſe the plane may recline more or leſſe than the Equinoctial, and conſequently their poles doe fall above or under the poles of the world, and the hour-lines doe likewiſe differ from the former.

The firſt variety is of a polar plane, or a plane reclining juſt to the Equinoctial, the projection whereof is ſo eaſie, that nothing need be ſaid of it, for a circle being divided into 24 equal parts, and lines drawn from the center to thoſe diviſions, ſhall be the meridians and hour circles, the ſtile or Gnomon, muſt be a ſtraight pin or wier perpendicular to the center, and the Dial is finiſhed.

The ſecond variety is of a *North* plane reclining leſs than the Equator, and might have been repreſented in the horizontal projection by a circle drawn like the Equinoctial, but between the Zenith and the Equinoctial circle; to project the hour-lines upon this plane, adde the reclination which ſuppoſe to be 25 deg. unto the complement of the poles elevation, that is in our latitude of *London* 38 $\frac{1}{2}$, their ſum is 63 $\frac{1}{2}$ the height of the pole above the plane, which known the hour-lines, and other neceſſaries may be projected, as in the Horizontal Dial hath been ſhewed.

The third varieties, is of a *North* plane reclining more than the Equator, and might have been represented in the Horizontal projection also, by a circle drawn like to the Equinoctial; but between the *South* intersection of the Meridian with the plane and the Equinoctial circle, to project the hour-lines upon this plane, you must consider, that because the Equator cutteth the Axis of the world at right angles, all planes that are parallel thereunto, have the height of the stile full 90 degrees above the plane: and by how much any plane reclineth from the Zenith, more than the Equator, by so much less than 90 is the height of the stile proper to it, and therefore if you adde the reclination to the distance between the pole and the Zenith, the complement of their sum is the poles elevation, as if the reclination from the Zenith were 70 degrees, and the distance between the pole and the Zenith 38 $\frac{1}{2}$ as before, there sum is 108 $\frac{1}{2}$ whose complement to a Semicircle is 71 $\frac{1}{2}$ the height of the pole above the plane: which being thus obtein'd, the hour-lines upon this Dial may be also projected, as hath been shewed in the Horizontal.

Lastly, as all other planes have two faces respecting the contrary parts of the Heavens; so these recliners have opposite sides, looking downwards to the Nadir, and they are called incliners, and may be therefore projected by the same rules, or rather they are both projected at once; for a *North* recliner more than the Equinoctial, is also a *South* incliner, and so of the rest.

CHAP.

CHAP. XII.

To project the hour-lines upon a declining reclining, or declining inclining plane.

Declining reclining planes have the ſame varieties that are in the *North* and *South* recliners; for either the declination may be ſuch, that the reclining plane will fall juſt upon the pole, and then it is called a declining Equinoctial; or it may fall above or under the pole, and then it is called a *South* declining *Eaſt* and *Weſt* recliner: On the other ſide, the declination may be ſuch, that the reclining plane ſhall fall juſt upon the interſection of the Meridian and Equator; and then it is called a declining polar; or it may fall above or under the ſaid interſection, and then it is a *North* declining *Eaſt* and *Weſt* recliner.

The three varieties of *South* recliners may be repreſented upon the plane of the Horizon by three circles, one falling between the Zenith and the pole of the world; another juſt upon the pole; and a third between the pole and the Horizon: and the particular pole of each plane is ſo much elevated above the Horizon as the plane it ſelf reclines from the Zenith. To fit a reclining plane juſt to the pole, to any declination given, or any reclination given, to find the declination proper to it, as hath been ſhewed in the 11 *Probl. of the 2 chap. of the 2 Part* of my Mathematical Inſtitution, it ſhall ſuffice to ſhew here how the hour-lines may be projected upon a declining reclining plane in any latitude the declination and reclination being given.

In

In the latitude of London, where the North *pole is elevated 51 ½, let it be required to project the hour-lines upon a* South *plane declining* East 30 *deg. reclining* 20 *deg.*

Fig. 16. Having drawn the fundamental circle, and crossed it with Diameters as before, set off the reclination 20 degrees from B unto C, and from D unto F, a ruler laid from W to C, shall cut the Diameter B A D in Z in the zenith, and a ruler laid from W to F shall cut the same Diameter extended in N the Nadir, so have you two points given through which the meridian must pass; a third point will easily be had if first you draw the Horizon, which differeth according to the reclination, the reclination therefore being set from E to G, a ruler laid from W to G, will cut the Diameter B A D in H, so have you the three points E H W to describe the horizon by; or the reclination being set from B to K, and from K to L, a ruler laid from W to L will cut the Diameter B A D in the center of the Horizontal circle E H W: then set the planes declination given from D to M (that is westward because the plane declineth *East*) a ruler laid from Z to M shall cut the Horizon in Q, which is the third point by which to describe the meridian N Q Z, the center whereof will be in the point R, and a ruler laid from R to A the center of the primitive circle will cut the Horizon in S the pole of the meridian, and a line drawn thereby, if extended beyond the primitive circle will give the point T the *West* point of the Horizon, and Equinoctial. A ruler laid from S the pole of the meridian to Z the Zenith, will cut the primitive circle in V, make V X equal to the distance of the *North* pole from the Zenith 38 ½, a ruler laid from S to X, shall cut the Merian in Y the *North* pole, a ruler d from S to N will cut the primitive circle in *a*, make *a b* equal to V X, a ruler laid from S to *b* will cut the meridian in *d* the *South* pole. And to describe the Equinoctial, we have given S and T the *East* & *West* points

points of the Horizon through which it must pass, and because it must also cut the meridian at Right angles, and be 90 degrees from either pole, a ruler laid from S the pole of the meridian to *y* the *North* pole, will cut the primitive circle in X, make W *e* equal to B X, which will be also equal to D *b*, if there were no former errour in the work, a ruler laid from S to *e* will give the point *f* in the meridian; which is the third point by which to describe the Equinoctial if you think fit.

A line drawn from *y* to *d* is the Axis of the world, which being bisected, a right line drawn at right angles in the point of bisection shall be the tangent line in which the centers of the meridians or hour-circles will fall, and if rightly drawn will pass through the center of the meridian.

If you desire the centers of the meridians in the tangent-line, set one foot of your Compasses in that pole which is elevated above the plane, and take the distance of the intersection of the tangent line with the Axis as at *h* (or any other distance) and therewith describe a circle, where a ruler laid from the center of this circle to the center of the meridian shall intersect this new inscribed circle as here at *m*, begin to divide the said circle into 24 equal parts, as hath been shewed before, a ruler laid from the elevated pole to these equal divisions shall cut the tangent line in the center of the hour-circles.

Or if the intersections of the meridians with the plane be onely required, having drawn the Equinoctial by the three points before given, a ruler laid from the elevated pole above the plane, to the point of intersection of the Equinoctial with the meridian, will cut the plane in *l*, where you must begin to divide the plane into its requisite parts, a ruler laid from the center of the plane to those equal parts, will cut the Equinoctial into its unequal parts, and a ruler laid from the elevated pole to those unequal parts of the Equi-

 noctial,

noctial, will cut the plane in the points through which the meridians must pass, and right lines drawn from the Center of the plane, through those points shall be the hour-lines desired; and F *d* is the height of the stile.

By these directions and the former Examples the description of the hour-lines is so easie, that I thought it not necessary to express them in this Scheme, but have left the perfecting of this Dial, and the description of the other variety of Declining recliners, to the practice of the Ingenious Reader; for all which the directions already given, (if well understood,) will be sufficient, and not sufficient onely for the projecting of Dials, but for the projecting of the several circles of the Globe, upon the plane of any great circle which shall be propounded.

FINIS.

ERRATA

After these words in the 3 line of the 12 page, perpendicular B F, *should follow*, equal to half A B. Pag. 12 lin. 3 for perpendicular B F, *read* perpendicular B F $= \frac{1}{2}$ A B.

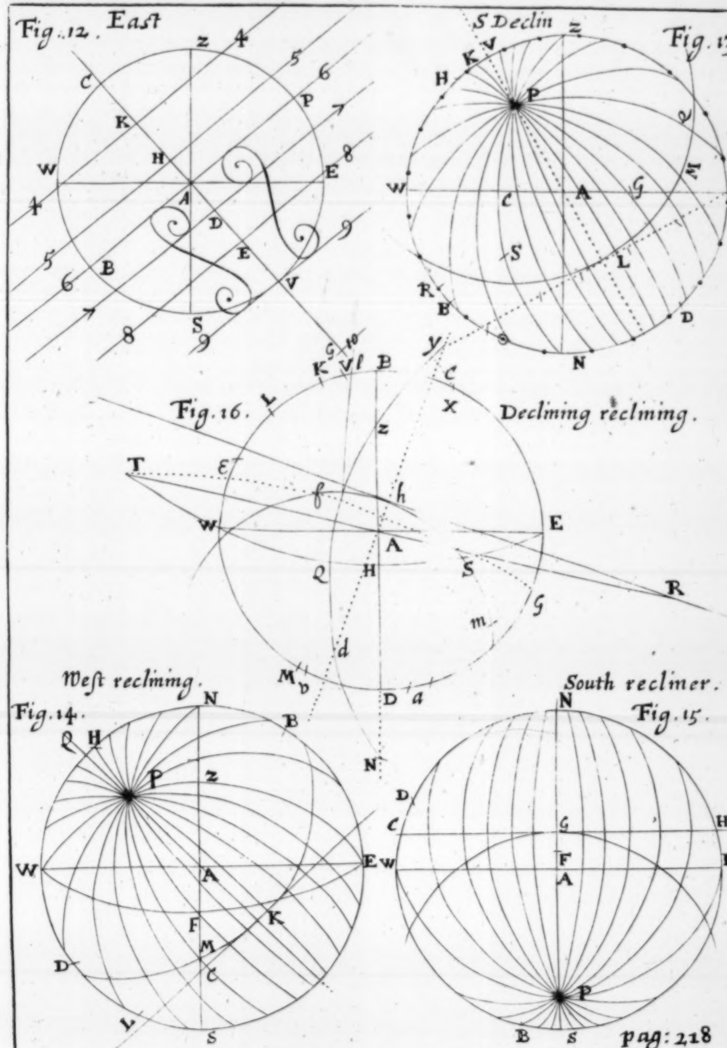
Fig. 12. East
S Declin
Fig. 13.
Fig. 16.
Declining reclining.
West reclining.
Fig. 14.
South recliner.
Fig. 15.
pag: 218

POST-SCRIPT.

THe *Problems of Sailing mentioned in the* 17 *&* 18 *Chapters of the first Part, may be more easily performed, if thou be furnished with* Blank Charts *exactly drawn, for which purpose we have caused such to be made in Imperial Paper, both for* Plain *and for* Mercators Sailing, *and are to be sold by* ROBERT HORNE *at the* Turks-head *in* Cornhil.

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